HOW FAST CAN YOU WATCH?
by Joe Dan Austin in the *Mathematics Teacher*

Have you ever been riding in a car and noticed that as you pass an object it is often not possible to follow it continuously with your eyes? Of course, it is easy to follow far away objects as they seem to move past slowly. However, when an object is too close to the car, it seems to go by too rapidly for the eyes to follow. Consider the mathematics of this phenomenon (this is a RELATED RATE PROBLEM).

In the figure below, a car is shown on a straight road. Assume the car travels at a constant speed r. The object being watched, a tree, is a distance s from the road. The distance a car must travel in order to pass by or be opposite the tree is *x*. A right triangle is drawn. Using the Pythagorean Theorem, find the length of the hypotenuse: __________________________

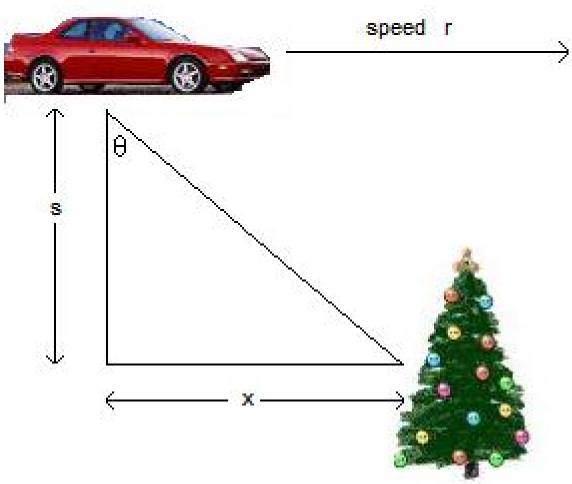
Let θ be the angle shown. As the eye follows the tree, the rate that the eye moves is $d\theta / dt$, the change in θ with respect to time *t*. As the car passes the tree, the speed the eye must move in order to follow the tree is $d\theta / dt$, for $x = 0$.

To explain why it is not always possible to follow an object with your eyes, we first examine $d\theta / dt$, the rate your eyes would have to move. From the right triangle below,

(1) $\cos\theta = \text{\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_}$

Differentiating implicitly with respect to *t*, we find

(2) ____



From the triangle, we know that

(3) $\sin\theta = \text{\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_}$

We also know that

(4) $r = \frac{\underline{\hspace{4cm}}}{dt}$

Substituting (3) and (4) in (2), and then solving for $d\theta/dt$, we get

(5) $\frac{d\theta}{dt} = \underline{\hspace{7cm}}$

Since we assumed that both r and s are constant, the maximum for $d\theta/dt$ occurs when x = O:

(6) So $d\theta/dt$ = ____________________________

Now we can explain why it is difficult to follow some objects as you pass them. As the rider watches an object that is close to the car, the value of *s* is small. The specific value of *s* can be made as small as we wish by choosing an object sufficiently close to the road to watch. From (6), this means that the maximum rate *r/s* that the eye must turn in order to follow the object can be made larger than any finite value. Specifically, *r/s* can be made larger than the rate that the eyes can move.

Let's estimate how near an object can be to the road and still be followed with the eyes when the car is driven at the current speed limit of 55 miles per hour (80.67 feet per second). To determine how fast your eyes can move and follow an object, hold your arm straight out and watch your thumb. Move your arm and thumb (but not your head) as fast as you can and still follow the thumb continuously with your eyes. Move the thumb a full 180 degrees or π radians. Suppose you can do this in 1/2 second. The maximum angle movement in radians for your eyes would be

(7) $\frac{\underline{\hspace{4cm}}}{}$ = ________________ radians per second.

Using (6), you can follow the object as you pass it if

(8) *r/s* $\leq$ ____________________

For a car traveling 55 mph, *r* = 80.67 feet per second. Using this value in (8) and solving for s implies that you can follow the object as you pass by it if the distance s of the object from the road satisfies

(9) $s \geq \frac{r}{2\pi}$ = _________________ feet

Thus, if the object is closer than _______ feet, your eyes can NOT continuously follow it as you pass it.

Try this experiment the next time you are riding (NOT driving). When traveling at 55 mph, how close can an object be to the road before you are unable to follow it with your eyes? How good is the estimated value in (9) for you?