A View of Mathematics Through A Camera Lens

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Consider the operation of a camera lens:

$$\text{f-stop} = \frac{\text{focal length of lens}}{\text{aperture diameter}}$$

$$f = \frac{l}{d}$$

Observe that as d decreases (the lens closes), then f increases.
Now the amount of light admitted depends on the area of the aperture, which in turn depends on d.

$$A = \pi r^2 = \pi \left(\frac{d}{2}\right)^2 = \frac{\pi}{4} d^2$$

and since $f = \frac{l}{d}$, $d = \frac{l}{f}$ and $A = \frac{\pi}{4} \frac{l^2}{f^2}$

For a standard 50 mm, l = 50 mm, so for f = 1.4, $A_{1.4} \approx 1001.28$ and for f = 2, $A_2 \approx 490.87$.
Observe that $A_2 \approx \frac{1}{2} A_{1.4}$. This pattern will continue. The area is halved at each successive f-stop.
Now if A is halved each time, then f^2 is doubled. Hence f is multiplied by $\sqrt{2}$. Thus, the f-stop numbers represent this progression:

$$1, \quad \sqrt{2}, \quad 2, \quad 2\sqrt{2}, \quad 4, \quad 4\sqrt{2}, \quad 8, \quad 8\sqrt{2}, \quad 16, \quad 16\sqrt{2}, \quad 32, \quad 32\sqrt{2},$$

which is a geometric sequence!

The amount of light that enters a camera is determined by the f-stop setting of the lens. The commonly used f-stop numbers are 1, 1.4, 2, 2.8, 4, 5.6, 8, 11, 16, 22, 32, and 45.