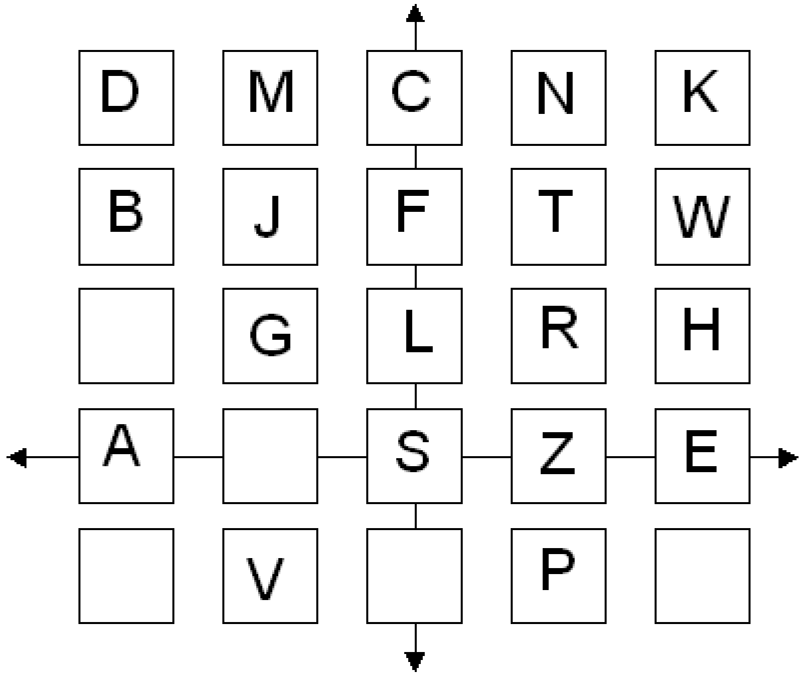
Algebra / Geometry

Class, Take Your Seats *Answer Key*



CLUE Worksheet

CLUE	NAME	Possible Ordered Pairs
1		
2	Archimedes	(2, 0), (-2, 0)
3	Boole	(-2, 2)
4	Cauchy	(0, 3)
5	Dirichlet	(2, 1), (-2, 3)
6	Euler	(2, 0), (-2, 2)
7	Fibonacci	(-2, 3), (0, 2), (2, 1)
8	Galois	(-1, 1)
9	Hilbert	(2, 1)
10	Jacobi	(-1, 2)
11	Kepler	(2, 3)
12	Leibniz	(-1, 3), (0, 1), (1, -1)
13	Maclaurin	(0, 2), (-1, 3), (1, 1) (2, 0)
14	Napier	(1, 3)
15	Pascal	(1, -1,), (-1, 3)
16	Russell	(1, 1)
17	Steno	(0, 0)
18	Thales	(-2, 2), (-1, 2), (0, 2), (1, 2), (2, 2)
19	Viete	(1, -1), (2, 0), (1, 1), (0, 2) (-1, -1), (-1, 1), (-2, 0)
20	Wallis	(2, 2), (1, 3)
21	Zorn	(0, 1), (-1, 2), (-2, 3), (1, 0), (2, 1)

Solutions to Problems:

CLUES:

2. A line of slope m = 3 intersects the x-axis at point A and the y-axis at point B. The point O is the origin and the area of triangle AOB is 6 square units. Archimedes sits at point A.

   We will denote the x-intercept by A(k, 0).
   Since the slope of the line is 3, the equation of the line is y = 3x +b from which we can find the y-intercept.
   Substituting the coordinates of A into the equation gives 0 = 3k + b so b = - 3k.
   The area of the triangle is $\frac{(AO)(BO)}{2} = \frac{k(3k)}{2} = 6$.
   Solving for k, we obtain $3k^2 = 12$, so $k = \pm 2$.

3. A circle is tangent to the x-axis at (-2, 0) and tangent to the y-axis at (0, 2). Boole sits at the coordinates of the center of the circle.

   Because the radii are perpendicular to the axes, the coordinates of the center are (-2, 2).

4. A circle intersects the x-axis at (- 4, 0) and (4, 0) and intersects the y-axis at (0, -2) and (0, 8). Cauchy sits at the coordinates of the center of the circle.

   The center can be found by taking the perpendicular bisector of the segment joining (- 4, 0) and (4, 0) and the perpendicular bisector of the segment joining (0, -2) and (0, 8), and seeing where they intersect (0, 3).

5. A (4, 5) and B (0, 7) are two consecutive vertices of square ABCD. Dirichlet is seated at one of the other vertices of the square (C or D).

   Consecutive sides of a square are perpendicular to each other, so their slopes are negative reciprocals. From A to B, one travels 4 units to the left and 2 units up. So from B to C, there are two possibilities: one must travel 2 units to the left and 4 units down (-2, 3) or one must travel 2 units to the right and 4 units up (2, 11). The latter does not give coordinates for C and D that lie in the domain. So, C must be (-2, 3) and then D would be (2, 1).

6. A (-1, -1) and S (1, 3) are opposite vertices of square RAMS. Euler sits at one of the other vertices (R or M).

   The midpoint of the segment SA is Q (0, 1). By rotating the diagonal SA 90 degrees around the point Q, we obtain the diagonal RM. Note that diagonal SA has slope 2, so diagonal RM has slope –1/2. To get from Q to S, we moved up 2 and across 1. To get to the other vertices, we move from Q (0, 1) down 1 and to the right 2 to get M (2, 0), and up 1 and to the left 2 to get R (-2, 2).

7. A (-5, 2) and B (-3, 6) are two vertices of the isosceles triangle ABE with AE = BE. Fibonacci is seated at the coordinates of vertex E.

First find the midpoint of AB: $\left(\frac{-3+(-5)}{2}, \frac{6+2}{2}\right) = (-4, 4)$. The slope of AB is 2, so the slope of the perpendicular bisector of AB is -1/2. Therefore vertex E must lie on the line $y = -\frac{1}{2}x + 2$. The only points in our domain that lie on that line are (-2, 3), (0, 2), and (2, 1).

8. The points (-1, 0), (-1, 4), (3, 4), and (3, 0) form a square. A line whose x-intercept is (-3, 0) cuts the square into two regions of equal area. Galois sits at one of the points of intersection of the line and the square.

Let (-1, k) and (3, 4-k) be the points of intersection of the line and the square (draw a diagram to help see this). Then the slopes through (-3, 0), (-1, k) and (3, 4-k) must be equal, so: $\frac{k-0}{-1+3} = \frac{4-k}{3+3} = \frac{(4-k)-k}{3-(-1)}$. Solving the proportion, we get k = 1. Therefore, the two points of intersection are (-1, 1) and (3, 3), but only the former is in our domain.

9. The isosceles triangle ABD has vertices A (-3, 0) and B (1, - 4), and AD = BD. Hilbert is seated at the coordinates of D that will create a triangle ABD of area 12 square units.

The slope of AB is -1 and the midpoint of AB is (-1, -2).
So, the slope of the perpendicular bisector of AB (where point D must lie) is 1.

Therefore D must lie on the line y = x – 1.

Since the length of AB is $4\sqrt{2}$ units, the height must be $3\sqrt{2}$ units.

So, the distance from (-1, -2) to D (x, x-1) must equal $3\sqrt{2}$ units.

Use the distance formula to get

$$\sqrt{(x-(-1))^2 + ((x-1)-(-2))^2} = 3\sqrt{2}$$
$$\sqrt{(x+1)^2 + (x+1)^2} = 3\sqrt{2}$$
$$\sqrt{2(x+1)^2} = 3\sqrt{2}$$
$$|x+1| = 3, \quad \text{so} \quad x = 2 \text{ or } x = -4$$

Therefore, Hilbert sits at the point (2, 1) since that is the only point that satisfies the domain.

10. M (2, 6) is the midpoint of the segment AB with A on the line with equation y = 2x and B on the line with equation y = x + 3. Jacobi is located at the coordinates of either point A or point B.

Since A is on the line of equation y = 2x, we can write A (a, 2a).
Since B is on the line of equation y = x+3, we can write B (b, b+3).

Using the formula for the midpoint of a line segment, we write $\begin{cases} 2 = \dfrac{a+b}{2} \\ 6 = \dfrac{2a+b+3}{2} \end{cases}$

This system simplifies to $\begin{cases} a+b=4 \\ 2a+b=9 \end{cases}$

Subtracting the first equation from the second, we obtain a = 5 which gives b = -1.
Thus we have A (5, 10) and B (-1, 2), so Jacobi must sit at (-1, 2).

11. A circle has its center P on the line y = x + 1, passes through the point (-1, 3), and is tangent to the x-axis. Kepler sits at the center of this circle.

Since P, the center of the circle, is on line y = x + 1, we can write P (p, p+1).
The circle is tangent to the x-axis at a point B (p, 0), and
the radius of the circle is r = PB = p + 1.
The radius of the circle is also $\sqrt{(p+1)^2+(p+1-3)^2}$.
So,

$$p+1=\sqrt{(p+1)^2+(p-2)^2}$$
$$(p+1)^2=(p+1)^2+(p-2)^2$$
$$p=2$$

Therefore, Kepler must sit at (2, 3).

12. Leibniz sits on the line that is perpendicular to $y=\frac{1}{2}x+1$ and passes through the point (2, -3).

First notice that the line $y=\frac{1}{2}x+1$ has slope ½, so the line perpendicular to it must have slope -2. Starting at point (2, -3), go back 1 and up 2 to get to (1, -1), then go back 1 and up 2 to get to (0, 1) and go back 2 and up 1 to get to (-1, 3). Leibniz must sit at one of these three points.

13. Maclaurin sits on the line that is parallel to x + y = 2011 and passes through the point (-3, 5).

Note that the slope of the line x + y = 2011 is -1,
so the slope of a line parallel to it must also be -1.
Starting at (-3, 5), go to the right 1 unit and down 1 unit to get to (-2, 4).
Continue in this manner to get points (-1, 3), (0, 2), (1, 1), and (2, 0).

14. Line QU has x-intercept at (7, 0). Line XY is perpendicular to QU and has y-intercept at (0, 1). The two lines intersect at a point on the line y = 3x. Napier sits at this point of intersection.

The point of intersection is situated on the line y = 3x, so we can call its coordinates (c, 3c).

The slopes of QU and XY are given by $\frac{3c-0}{c-7}$ and $\frac{3c-1}{c-0}$, respectively.

Since the two lines are perpendicular, we have $m_{QU} = \frac{-1}{m_{XY}}$, which translates into

$\frac{3c-0}{c-7} = -\frac{c-0}{3c-1}$.

Dividing by c on both sides and simplifying, we get

$$9c - 3 = -c + 7$$
$$c = 1$$

So, Napier must sit at the point (1, 3).

15. The square ABCD has vertex A with coordinates A (-3, -3). The diagonal BD is located on the line with equation x + 3y = -2. Pascal sits at the coordinates of one of the other vertices B, C, or D.

Draw a diagram to help you visualize the problem.

Since the diagonals of a square are perpendicular, the line containing the diagonal AC has slope m = 3.

Therefore, the equation of the line AC is y – (-3) = 3 (x – (-3)).

The center of the square is found by intersecting BD and AC.

We have the system:

$$\begin{cases} DB: & x + 3y = -2 \\ AC: & 3x - y = -6 \end{cases}$$

Solving, we get x = -2 and y = 0.

Starting at (-2, 0), use the slopes to get B(1, -1), C (-1, 3), and D (-5, 1).

So, Pascal must sit at either (1, -1) or (-1, 3).

16. The points M (2, 4), N (0, 3), and P (3, 2) are the midpoints of the sides of the triangle ABC. Russell sits at the coordinates of one of the vertices of the triangle ABC.

Draw a diagram to help visualize the problem.

Since M and P are the midpoints of the sides BC and AC, respectively, MP is parallel to AC.

For similar reasons, we also have $MN \parallel AB$ and $NP \parallel BC$.

Therefore, $m_{BA} = m_{NM} = \frac{1}{2}$ and $m_{AC} = m_{MP} = -2$ and $m_{BC} = m_{NP} = \frac{-1}{3}$.

We can then write equations for the three lines:

$$\begin{cases} BC: \quad m = \frac{-1}{3}, \text{ passes through } (2,4) \quad y - 4 = \frac{-1}{3}(x-2) \\ BA: \quad m = \frac{1}{2}, \text{ passes through } (3,2) \quad y - 2 = \frac{1}{2}(x-3) \\ AC: \quad m = -2, \text{ passes through } (0,3) \quad y - 3 = -2(x-0) \end{cases}$$

This system simplifies to:

x + 3y = 14
x – 2y = -1
2x + y = 3

Solving simultaneously, we get A (1, 1), B (5, 3), and C (-1, 5).

So, Russell must sit at (1, 1).

17. Steno is seated on the graph of $x^2 + y^2 = 0$.

The graph has only one point (0, 0).

18. Thales is seated on the graph of $|y| = 2$.

Y must be equal to 2 or -2, so the graph is two parallel lines with slopes equal to zero, one passing through the point (0,2) and the other through (0, -2). Only the former is in our domain, so Thales must be at (-2, 2), (-1, 2), (0, 2), (1, 2), or (2, 2).

19. Viete is seated on the graph of $|x| + |y| = 2$.

Examine four cases corresponding to the four quadrants:

Case 1: If $x \ge 0, y \ge 0,$ the equation becomes $x + y = 2.$

Case 2: If $x < 0, y \ge 0,$ the equation becomes $-x + y = 2.$

Case 3: If $x < 0, y < 0,$ the equation becomes $-x - y = 2.$

Case 4: If $x \ge 0, y < 0,$ the equation becomes $x - y = 2.$

Combining the four we get a square with the vertices on the axes.

20. Wallis is seated on the graph of $|x+y|=4$.

We have two possibilities:

x + y = 4 or x + y = -4.

The graph is composed of two parallel lines.

The only points in our domain are (2, 2) and (1, 3).

21. Zorn is seated on the graph of $y=|x-1|$.

This absolute value curve forms a V with its vertex at (1, 0).

This can be seen by using the definition of absolute value to get the following:

If $x-1\geq 0$ or $x\geq 1$, then $y=x-1$.

If $x-1<0$ or $x<1$, then $y=-x+1$.

Graph the first equation for points whose x-coordinates are greater than or equal to 1, and graph the second equation for points whose x-coordinates are less than 1.

There are five points that are contained in our domain:
(0, 1), (-1, 2), (-2, 3), (1, 0), and (2, 1).