Factoring Techniques

by David Pleacher

I. Always remove from each term the highest common factor first.

A. Monomial factor

ex. $15x + 3 = 3 (5x + 1)$

B. Binomial factor

ex. $3 (x + 1) + y (x + 1) = (x + 1) (3 + y)$

II. Decide how many terms are left.

A. Two terms

1. Sum of two squares: they are prime (see note below)

2. Difference of two squares $A^2 – B^2 = (A – B) (A + B)$

ex. $9x^2 – 4y^2 = (3x – 2y) (3x + 2y)$

3. Sum of two cubes $A^3 + B^3 = (A + B) (A2 - AB + B2)$

ex. $x3 + y3 = (x + y) (x^2 - xy + y^2)$

4. Difference of two cubes $A^3 – B^3 = (A – B) (A^2 + AB + B^2)$

ex. $8a^6 - w^3 = (2a^2 -w) (4a^4 + 2a^2w + w^2)$

B. Three terms

1. Trinomial where a combination is found

ex. $2x^2 -9xy + 4y^2 = (2x -y) (x -4y)$

a. Perfect square trinomial

ex. $x^2 + 2xy + y^2 = (x + y)^2$

C. Four or more terms

1. Group by two's so as to take out a common factor.
   ex. $2ac - 2bc + ad - bd = 2c(a - b) + d(a - b)$
   $= (a - b)(2c + d)$

2. Group three terms and one term to make a difference of two squares.
   ex. $a^2 + 2ab + b^2 - c^2 = (a + b)^2 - c^2$
   $= (a + b + c)(a + b - c)$

III. Other suggestions

A. Factors must be prime.

B. Terms may be rearranged, but no signs may be changed except in the following:

ex. $2(c - d) + k(d - c) = 2(c - d) - k(c - d)$
$= (c - d)(2 - k)$

Note about the Sum of Two Squares:
Steve Schwartzman of Austin, Texas noted that there are some expressions of the sum of two squares which can be factored over the real numbers and he sent in the following example:

$$4x^4 + y^4 = \left(2x^2 + y^2 - 2xy\right)\left(2x^2 + y^2 + 2xy\right)$$