Numbers and Mysticism

Consider D_{284} = {1, 2, 4, 71, 142, 284}.
The proper divisors of 284 are all the divisors except the number itself.
Since 1 + 2 + 4 + 71 + 142 = 220, the sum of the proper divisors is 220.
What is the sum of the proper divisors of 24? _____

Now consider the following definitions:

1. A number is called <u>abundant</u> if the sum of its proper factors exceeds the number.
   Example: 24 is abundant since 1 + 2 + 3 + 4 + 6 + 8 + 12 = 36.

   Find all the abundant numbers between 1 and 20 inclusive: _____

2. A number is called <u>deficient</u> if the sum of its proper factors is less than the number.
   Example: 8 is deficient since 1 + 2 + 4 = 7.

   Find all the deficient numbers between 1 and 20 inclusive: _____

3. A number is called <u>perfect</u> if the sum of its proper factors equals the number.
   Example: 6 is perfect since 1 + 2 + 3 = 6.

   Find a perfect number between 20 and 30: _____

   Show that 496 is a perfect number: _____

4. Two numbers are <u>amicable</u> if each is the sum of the proper divisors of the other.

   Example:
      D_{284} = {1, 2, 4, 71, 142, 284}, and
         1 + 2 + 4 + 71 + + 142 = 220.
      D_{220} = {1, 2, 4, 5, 10, 11, 20, 22, 44, 55, 110, 220}, and
         1 + 2 + 4 + 5 + 10 + 11 + 20 + 22 + 44 + 55 + 110 + 220 = 284.
       So, 220 and 284 are amicable.

   Show that 1184 and 1210 are amicable numbers: _____

   _____

SHEET 2

Numbers and Mysticism

Much of the mysticism surrounding numbers dates back to the time of Pythagoras (500 B.C.). Special meanings were given to numbers depending on whether they were perfect, abundant, deficient, or amicable. For example, two people bearing numbers that are amicable would seal a perfect friendship between them. Persons or objects associated with deficient numbers were considered inferior, with abundant numbers above average, and with a perfect number -- well, you can imagine. Below you will find a number assigned to each letter of the alphabet. Using this assignment, you can decide whether a person or object is associated with a deficient, abundant, or perfect number.

```
A - 1        H -  9       O - 47       V - 37
B - 2        I - 10       P - 64       W - 66
C - 4        J - 13       Q - 29       X - 87
D - 5        K - 18       R - 85       Y - 90
E - 6        L - 33       S - 80       Z - 100
F - 7        M - 15       T - 81
G - 8        N - 17       U - 14
```

Example: The name WILLIAM

W + I + L + L + I + A + M = 66 + 10 + 33 + 33 + 10 + 1 + 15 = 168

D_{168} = {1, 2, 3, 4, 6, 7, 8, 12, 14, 21, 24, 28, 42, 56, 84, 168}

1 + 2 + 3 + 4 + 6 + 7 + 8 + 12 + 14 + 21 + 24 + 28 + 42 + 56 + 84 = 312

The number is abundant since the sum of the proper divisors (312) is
          greater than the number (168).

Use the procedure in the example to analyze the following:

1. The name Michael _____

2. The name Kathryn _____

3. The dance Cha Cha _____

4. Your name _____

5. The name John Handley _____

6. The name Pleacher _____

SHEET 3

Numbers and Mysticism

To compare two persons or objects, use the following rules:

1. Amicable numbers indicate a perfect relationship.

2. If the numbers are both abundant or both deficient, a good relationship exists.

3. If one number is abundant and the other is deficient, a poor relationship exists.

Example: SCHOOL and STUDENT

S + C + H + O + O + L = 80 + 4 + 9 + 47 + 47 + 33 = 220

S + T + U + D + E + N + T = 80 + 81 + 14 + 5 + 6 + 17 + 81 = 284

On Sheet 1, the numbers 220 and 284 were shown to be amicable --
No wonder they get along so well together!

Try the following:

1. Fire and Water _____

2. Peanut Butter and Jelly_____

3. The word President and the name GEORGE W. BUSH _____

4. Your name and your desired profession _____

The way the letters of the alphabet are assigned is arbitrary.
Try your hand at making up an assignment so that you and a friend will be amicable (like 220 and 284).

A –	H –	O –	V –
B –	I –	P –	W –
C –	J –	Q –	X –
D –	K –	R –	Y –
E –	L –	S –	Z –
F –	M –	T –	
G –	N –	U –	