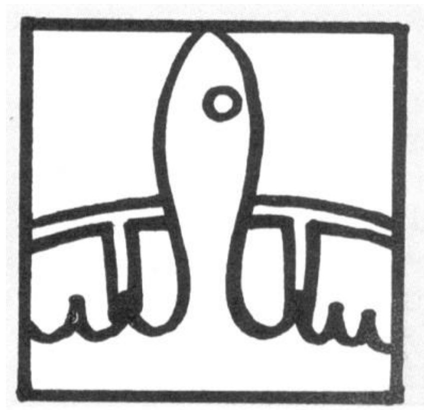
Turvy #12   Conics & Their Applications

A Puzzle by David Pleacher



Back in 1953, Roger Price invented a minor art form called the Droodle, which he described as "a borkley-looking sort of drawing that doesn't make any sense until you know the correct title."   In 1985, Games Magazine took the Droodle one step further and created the Turvy. Turvies have one explanation right-side-up and an entirely different one turned topsy-turvy. The Turvy above was created by J. Zimmerman of Cleveland, Ohio and published in Games Magazine in May 1986.

Here is the title right-side-up:

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__  __  __  __  __,     __  __  __
15   1  11  12   3       8   1  16

__  __   __  __   __   __  __  __  __  __     __  __  __  __  __
 9  16    6  14    5    6  12  10  17   2     13   1  11   4  16
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Here is the title upside-down:

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__  __  __     __  __     __  __  __  __  __  __  __
12  11  17     10  17     15  11  17   7  11   4  15

__  __  __  __  __  __  __     __     __  __  __  __
14  10  17   7  10  17   2     11      7  10  12  16
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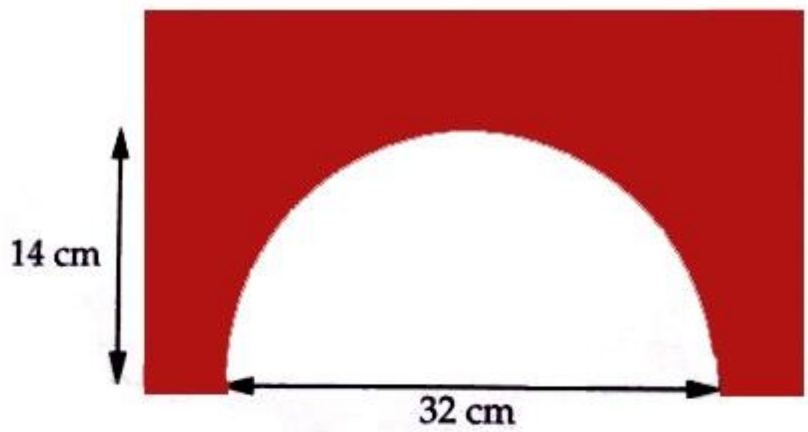
To determine the titles to this turvy, solve the 17 problems in this puzzle about conics and their applications. Then replace each numbered blank in the puzzle with the letter corresponding to the answer for that problem.

Problems:

___ 1. Write an equation for the ellipse with vertices (4, 0) and (–2, 0) and foci (3, 0) and (–1, 0).

___ 2. The diagram below shows the cross-section of a bridge for a model train. The highest point of the arch is 14 cm, and the span at the base of the arch is 32 cm. Find the equation of the elliptical arch of the bridge. Let the center of the ellipse be at the midpoint of the 32 cm segment, and call this the origin.



___ 3. Write the equation of the parabola for which y = 1 is the directrix and F(3, –2) is the focus.

___ 4. A "whispering room" is one with an elliptically-arched ceiling. If someone stands at one focus of the ellipse and whispers something to his friend, the dispersed sound waves are reflected by the ceiling and concentrated at the other focus, allowing people across the room to clearly hear what he said. Suppose such gallery has a ceiling reaching twenty feet above the five-foot-high vertical walls at its tallest point (so the cross-section is half an ellipse topping two vertical lines at either end), and suppose the foci of the ellipse are thirty feet apart.
What is the height of the ceiling above each "whispering point"?

___ 5. Determine the equation of a circle that passes through the points (3, –2), (5, 3), and (–1, 9).

___ 6. The Golden Gate Bridge is a suspension bridge in San Francisco, California. The towers are 1280 meters apart and rise 160 meters above the road. The cable just touches the sides of the road midway between the towers. What is the height of the cable 200 meters from a tower?

___ 7. Determine the equation of the ellipse with center at (1, 3), major axis = 10 and parallel to the X-axis, and semi minor axis = 3.

___ 8. Determine the focus of the parabola $y = 2x^2 - 3x + 5$.

___ 9. A narrow arch supporting a stone bridge is in the shape of half an ellipse and is 24 meters long and 8 meters high. A person standing at one focus of the ellipse throws a rubber ball against the arch. No matter what direction the ball is thrown, it always bounces off the arch once and strikes the same point on the ground (the other focus). How far apart is the person throwing the ball from the point on the ground at which the ball strikes?

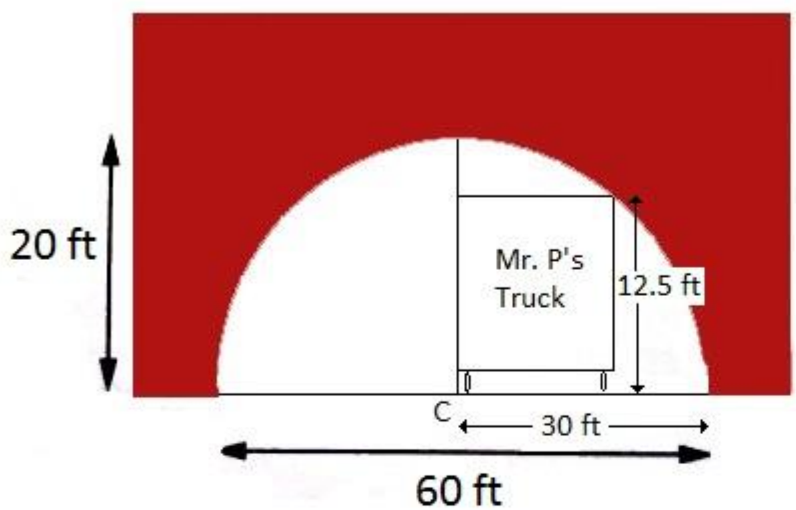
___ 10. Determine the equation of the hyperbola with center at (1, 3), transverse axis = 8 and parallel to the X-axis, and semiconjugate axis = 3.

___ 11. Suppose a satellite is in an elliptical orbit, with major axis = 8840 miles and minor axis = 8832 miles, and with the center of the Earth being at one of the foci of the ellipse. Assuming the Earth has a radius of about 3960 miles, find the lowest and highest altitudes of the satellite above the Earth.

___ 12. Determine the intersection points of the circle $x^2 + y^2 = 40$ and the hyperbola $xy = 12$.

___ 13. Determine the equations of the circle(s) that pass through (0, –3), with radius $r = \sqrt{5}$, and with centers on the angle bisector of the first and third quadrants.

___ 14. An arch of a bridge over a highway is semi-elliptical in shape and 60 feet across. The highest point of the arch is 20 feet above the highway. A truck that is 12 feet 6 inches tall wishes to pass through the tunnel. If one side of the truck is on the center stripe, what is the maximum width of the truck that can fit under the arch?



___ 15. Determine the vertex of the parabola $4y^2 - 16x - 20y - 7 = 0$.

___ 16. Determine the point(s) of intersection of the circle $x^2 + y^2 = 25$ and the line $x + 2y = 10$.

___ 17. Determine the coordinates of the foci of the ellipse

$$25x^2 + 16y^2 + 100x - 32y - 284 = 0.$$

Answers:

A. The minimum is 272 miles and the maximum is 648 miles.
B. The minimum is 3608 miles and the maximum is 4232 miles.
C. $(x - 1)^2 + (y - 3)^2 = 144$
D. $\frac{(x-1)^2}{25}+\frac{(y-3)^2}{9}=1$
E. (0, 5) and (4, 3)
F. 23.42 feet
G. $\frac{x^2}{256}+\frac{y^2}{196}=1$
H. $\frac{(x-1)^2}{9}+\frac{y^2}{5}=1$
I. $\frac{(x-1)^2}{16}-\frac{(y-3)^2}{9}=1$
J. (5, 2), (–5, –2), (2, 5), and (–2, –5)
K. 22 feet
L. 21 feet
M. (6, 2), (–6, –2), (2, 6), and (–2, –6)
N. (–2, 4) and (–2, –2)
O. $7x^2+7y^2+19x-37y-222=0$
P. 17.89 meters
Q. (1, 4)
R. 75.625 meters
S. $\left(-2,\frac{5}{2}\right)$
T. $\left(\frac{3}{4},4\right)$
U. $(x-3)^2=-6\left(y+\frac{1}{2}\right)$
V. $(x-1)^2+(y-1)^2=5$ and $(x+2)^2+(y+2)^2=5$
W. $(x+2)^2+(y+2)^2=5$ and $(x+1)^2+(y+1)^2=5$
X. $(-2,2)$
Y. $(x+3)^2=6\left(y+\frac{1}{2}\right)$
Z. None of the above