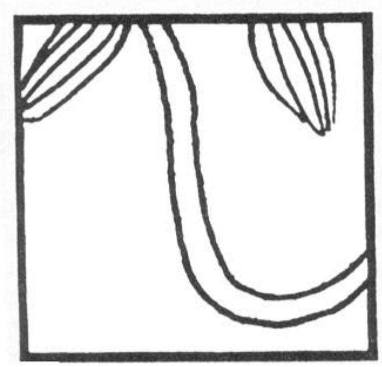
Turvy #13 Challenging Precalculus Problems

A Puzzle by David Pleacher



Back in 1953, Roger Price invented a minor art form called the Droodle, which he described as "a borkley-looking sort of drawing that doesn't make any sense until you know the correct title." In 1985, Games Magazine took the Droodle one step further and created the Turvy. Turvies have one explanation right-side-up and an entirely different one turned topsy-turvy. The Turvy above was created by the *Games* editors and published in *Games* in May 1986.

Here is the title right-side-up:

```
__     __ __ __ __ __ __ __ __ __     __ __ __
20      8  9 12 19 20  1 13 15  3      8 20  7

__ __ __ __ __ __     __ __ __ __ __ __ __ __ __
15 20 19 17  7 16     12 18 20 16 13 15 19 19 17
```

Here is the title upside-down:

```
__     __ __ __ __     __ __ __ __     __ __ __ __ __ __ __ __,
20     16 17 11  4     10 17 19 13     18 17 16 19 20 17  4 12

__ __ __ __ __ __ __ __     __ __ __ __
12  2 17 18 18 17  7 16     11  5 18 15
```

To determine the titles to this turvy, solve the 20 problems. Then replace each numbered blank in the puzzle with the letter corresponding to the answer for that problem.

Problems:

___ 1. Determine the smallest value of x satisfying the equation $|x|^2 + |x| - 6 = 0$.

___ 2. If $f(x) = x^3 + 3x^2 + 4x + 5$ and $g(x) = 5$

Then $g(f(x)) =$

___ 3. Determine the real number k for which the solution set of $|kx + 2| < 6$ is the open interval (– 1, 2).

___ 4. If $\log_8 M + \log_8\left(\frac{1}{6}\right) = \frac{2}{3}$, Then M =

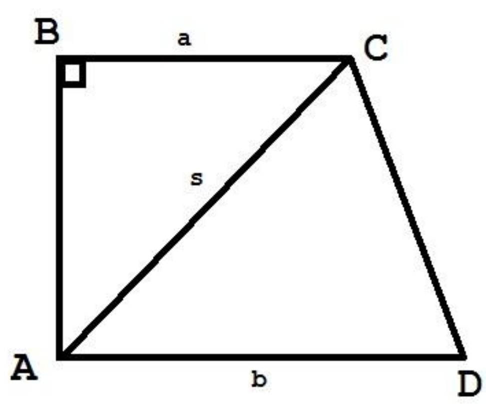
___ 5. If $f(x) = 2x^3 + Ax^2 + Bx - 5$ and if $f(2) = 3$ and $f(-2) = -37$

What is the value of A + B?

___ 6. A ball is dropped from a height of 1 meter. It always bounces to one-half its previous height. The ball will bounce infinitely but it will travel to a finite distance.
What is the distance?

___ 7. In Quadrilateral ABCD, $\overline{AB} \perp \overline{BC}$, $\overline{AD} \| \overline{BC}$, $m(\overline{BC}) = a$, $m(\overline{AC}) = s$, $m(\overline{AD}) = b$,

Determine $m(\overline{CD}) =$



___ 8. Determine the smallest positive solution θ of the equation $2\cos^2\theta + 3\sin\theta = 0$.

___ 9. Determine the **exact** value of $\sin\left(Cos^{-1}\left(-\frac{4}{5}\right) - Tan^{-1}\left(-\frac{12}{5}\right)\right)$.

___ 10. The sum of the first 83 nonnegative powers of i is _____.

Hint: $i^0 + i^1 + i^2 + i^3 + \ldots + i^{82} =$

___ 11. If $8^x = 4$ and $5^{x+y} = 125$, Determine y.

___ 12. Determine the sum of the solutions of the equation $|x^2 - 16| = 9x + 6$.

___ 13. Determine the coefficient of x^4 in the expansion $(x-2)^7 =$

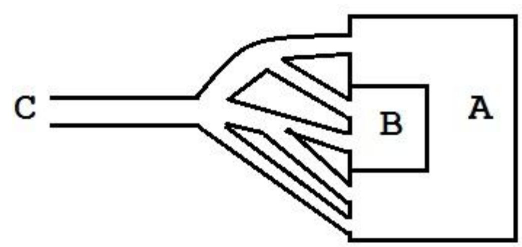
___ 14. Write $\cos(3\theta)$ in terms of $\sin\theta$ and $\cos\theta$.

___ 15. In a litter of 4 kittens, what is the probability that all are female?

___ 16. $(i^{17} + i^{10})^3 =$

___ 17. If $f(x) = \frac{x-1}{x}$ and $g(x) = 1 - x$, Then $f(g(x)) =$

___ 18. In the maze in the figure, Harry is to pick a path from C to either room A or room B. Choosing randomly at each intersection, what is the probability that Harry will enter room B?



___ 19. A bouncing ball loses ¼ of its previous height each time it rebounds. If the ball is thrown up to a height of 60 feet, how many feet will it travel before coming to rest?

___ 20. If $\sin 2x \sin 3x = \cos 2x \cos 3x$, determine the smallest positive value of x that satisfies the equation.

Answers (units are omitted because it would give some answers away):

A. 18
B. 36
C. -2
D. -4
E. $\frac{1}{16}$
F. $\frac{1-x}{x-1}$
G. 2i + 2
H. -280
I. $\frac{-x}{1-x}$
J. $\cos^2\theta - \sin^2\theta$
K. 5
L. 24
M. 210
N. $\sqrt{s^2+b^2-2ab}$
O. -1
P. $\frac{1}{3}$
Q. 3
R. $2\frac{1}{3}$
S. 12
T. 480
U. $-\frac{33}{65}$
V. $-\frac{63}{65}$
W. i
X. $\cos^3\theta - 3\sin^2\theta\cos\theta$
Y. $\sqrt{s^2-b^2}$
Z. None of the above