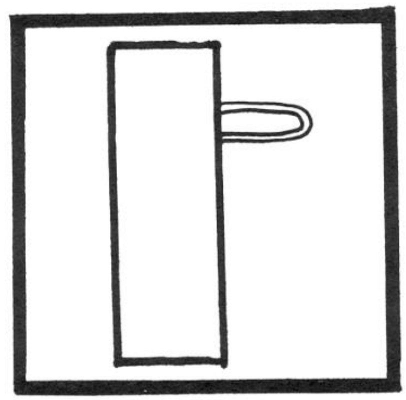
Turvy #14 Challenging Precalculus Problems

A Puzzle by David Pleacher



Back in 1953, Roger Price invented a minor art form called the Droodle, which he described as "a borkley-looking sort of drawing that doesn't make any sense until you know the correct title." This Droodle has one explanation right-side-up, a different one turned topsy-turvy, and a third interpretation on its side. The Turvy above was created by Roger Price and published in his book called *DROODLES*.

To determine the titles to this turvy, solve the challenging problems below. Then replace each numbered blank in the puzzle with the letter corresponding to the answer for that problem.

Here is the title right-side-up:

```
__ __ __    __ __ __ __ __ __ __    __ __ __ __ __ __ __ __
 3 17 19     6 11 17  8 10 19  7    12  2 18  3 16 18 19 20

        __ __    __ __ __ __ __    __ __ __ __ __
        10 19     6  9 18 19 20    16 18 18 12  9
```

Here is the title upside-down:

```
__ __ __ __ __ __    __ __ __ __ __ __ __    __ __ __ __ __ __ __ __
 3 10 14  7 20 12     6 11 17  8 10 19  7    12  2 18  3 16 18 19 20

        __ __    __ __ __ __ __    __ __ __ __ __
        10 19     6  9 18 19 20    16 18 18 12  9
```

If you turn the picture on its side counterclockwise, it is subject to a third interpretation:

```
__ __ __ __ __ __ __ __    __ __ __ __ __ __ __ __    __ __ __ __ __ __
14 20  1 20 17  4 20 14    12  2 18  3 16 18 19 20     6 11 17  8 20  2
```

Problems:

___ 1. Determine all values of x satisfying $|x|+|x+2|=4$.

___ 2. At what value of x does the function $f(x)=2-|x-3|$ have a maximum value?

___ 3. Graph the function $y=\dfrac{x+|x|}{2}$.

___ 4. Determine all solutions of $x^3 - 8 = 0$.

___ 5. Determine all solutions of $x^2 - 2ix + 8 = 0$.

___ 6. Determine the real values of x satisfying the equation $(2 + 5i)x - (3 + 4i)y = -1 - 6i$.

___ 7. If $\sin x+\cos x=\dfrac{1}{5}$ and $0\le x\le\pi$, Then $\tan x=$

___ 8. Determine the least positive value of Θ in degrees such that

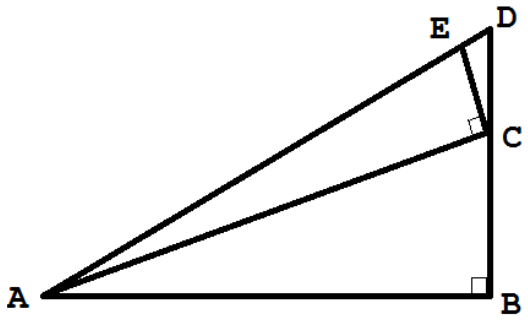
$$\sin^2\theta+\cos^2\theta+\tan^2\theta=\frac{4}{3}.$$

___ 9. If the Miami Heat lead the San Antonio Spurs 3 games to 2 in a 7 game playoff, and assuming the probability of the Heat winning any game against the Spurs is 3/5, what is the probability that the Spurs will win the playoff?

___ 10. Solve the inequality $|x-4|<5$.

___ 11. Solve the equation $2\sin^2\theta-\sin\theta=0$ for $0\le\theta<2\pi$.

12 – 14. In the figure at the right, $m\angle ABC = m\angle ACE = 90°$, AB = 4, BC = 3, & CE = 2.



___ 12. Determine AC.

___ 13. Determine AE.

___ 14. Determine AD.

___ 15. Determine $\sin\left(\text{Tan}^{-1}\frac{4}{3}\right)$.

___ 16. Determine $\text{Sin}^{-1}\left(\sin\frac{18\pi}{5}\right)$.

___ 17. Solve the equation $e^{2x} - 4e^{x} + 3 = 0$.

___ 18. Solve the equation $\log_3(x+5) - \log_3(x-7) = 2$.

___ 19. Solve the equation $x - 5\sqrt{x} = -6$.

___ 20. If $f(x) = \frac{2x-5}{x+4}$, Determine a formula for the inverse function $f^{-1}(x)$.

Answers (units are omitted because it would give some answers away):

A. 0, ln(3)

B. $\frac{-2\pi}{5}$

C. – 3, 1

D. $\frac{10\sqrt{29}}{7}$

E. $\frac{4x+5}{2-x}$

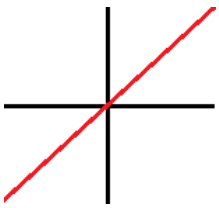
F. $10\sqrt{29}$

G. $\frac{-4}{3}$

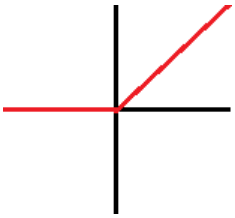
H. $\frac{21}{25}$

I. $-1<x<9$

J. –2i, 4i

K. 

L. $0, \pi, \frac{\pi}{6}, \frac{5\pi}{6}$

M. 

N. 4, 9

O. $\frac{17}{2}$

P. –2

Q. $\sqrt{29}$

R. 3

S. 2, $-1\pm\sqrt{3}i$

T. 5

U. –3, 8

V. $\frac{-4}{5}$

W. 60

X. $\frac{4}{5}$

Y. 30

Z. None of the above