Max / Min Problems for Review Calculus Mr. Pleacher

(1) German mathematician Johannes Regiomontanus (1436 -1 476) posed the following question:
From what position along a horizontal line can a statue best be viewed? If the spectator is too close, the statue will appear heavily foreshortened, thus distorting its size. If the spectator is too far away, it will simply be too small to see. An optimal distance for viewing the statue must exist.
For example, the height of the Statue of Liberty is 305 feet from the ground to the tip of the torch. If you are on a boat in the harbor and the deck is 10 feet below base of the statue, how far away should you be to view the statue at its optimal angle? What is the optimal angle?

Answers: 56.12 feet, angle is 69.79 degrees

---

(2) Given an isosceles triangle with equal sides of 3 inches in length. Determine the measure of the angle between the two equal sides which results in the largest area.

Answer: 90 degrees

---

(3) Construct a window in the shape of a semi-circle over a rectangle (like many stained glass windows in churches).
If the distance around the outside of the window is 12 feet, what dimensions will result in the rectangle having the largest possible area?

Answers: The rectangle's dimensions would be 2.33 feet by 3 feet, so the area would be 6.99 square feet.

---

(4) Ladder in the hall Problem
A non-folding ladder is to be taken around a corner where 2 hallways intersect at right angles. One hall is 7' wide and the other hall is 5' wide. What is the maximum length that the ladder can be so it will pass around the corner?

Answer: 16.89 feet

(5) During the Winter break, Matt E. Matics is enroute to his sister's house. Two policemen are stationed two miles apart along the road, which has a posted speed limit of 55 mph. The first clocks him at 50 mph as he passes, and the second policeman clocks him at 55 mph, but pulls him over anyway. When he asks why he got pulled over, the policeman responds, "It took you 90 seconds to travel 2 miles." Why is he guilty of speeding according to the Mean Value Theorem?

Answer: His average rate is 80 mph. There must have been at least one point over the interval where the instantaneous rate of change (velocity) must equal the average rate of change (average speed). Therefore, old lead foot Matt must have traveled 80 mph at least once, and the policeman can ticket him.

---

(6) The practice of shooting bullets into the air - for whatever purpose - is extremely dangerous.
Assuming that a hunting rifle discharges a bullet with an initial velocity of 3000 ft/sec from a height of 6 feet, answer the following questions:

(A) How high will the bullet travel at its peak? (B) At what speed will the bullet be traveling when it slams into the ground, assuming that it hits nothing in its path?

Answers: (A) 140,631 feet (it takes 93.75 seconds to reach the maximum height)
(B) 3000.064 ft/sec

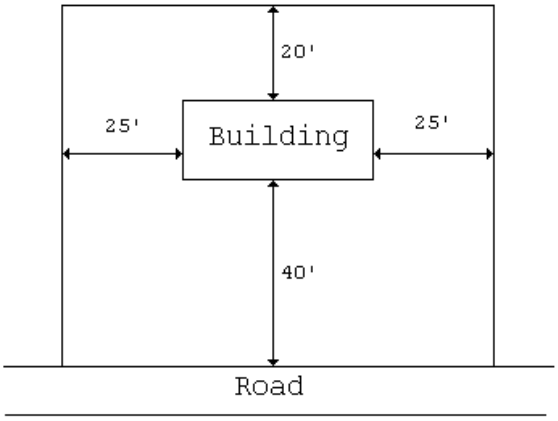
---

(7) To celebrate our 40th Anniversary in 2008, I am commissioning the construction of a
four-inch tall box made of precious metals to give to my wife. The jewelry box will have rectangular sides and an open top. The longer sides of the box will be made of gold, at
a cost of $300 per square inch; the shorter sides will be made of platinum, at a cost of $550 per square inch. The bottom will be made of plywood, at a cost of 2 cents per square inch.
What dimensions provide me with the lowest cost if I am adamant that the box have a volume of
50 cubic inches?

Answer: $C = 2400x + 4400y + xy(.02)$
Dimensions are 4.787 in x 2.611 in x 4 in.
(The lowest cost for the box will be $23,000 - do I still need to buy a card?)

---

(8) A builder is purchasing a rectangular plot of land with frontage on a road for the purpose of constructing a rectangular warehouse. Its floor area must be 300,000 square feet.
Local building codes require that the building be set back 40 feet from the road and that there be empty buffer strips of land 25 feet wide on the sides and 20 feet in the back. Find the overall dimensions of the parcel of land and building which will minimize the area of the land parcel that the builder must buy.



Answer: Dimensions of the building: 500' x 600'
Dimensions of the land: 550' x 660'