Applications of Derivatives

Analysis of Functions

- **Increasing / Decreasing**

  If $f'(x) > 0$ Then $f(x)$ is increasing.
  If $f'(x) < 0$ Then $f(x)$ is decreasing.
  If $f'(x) = 0$ Then $f(x)$ is constant.

- **Concavity**

  If $f''(x) > 0$, then $f(x)$ is concave up. (Mr. Happy Face)
  If $f''(x) < 0$, then $f(x)$ is concave down. (Mr. Frowny)

  Points of inflection occur when the concavity changes.
  Test: If there is a point of inflection, the second derivative is zero.
  BUT just because the second derivative is zero doesn't guarantee a point of inflection.

- **Relative Extrema**

  1st derivative test:
  Test points on each side of the critical points found by substituting in the first derivative.

  If the value of the derivative of the point to the left of the critical point is positive and the value of the derivative for the point to the right is negative, then the critical point is a relative maximum.

  If the value of the derivative of the point to the left of the critical point is negative and the value of the derivative for the point to the right is positive, then the critical point is a relative minimum.

  If the values of the derivative of the points to the left and the right of the critical point are the same (i.e., both positive or both negative), then the critical point is a point of inflection.

2nd derivative test:

Take the first derivative and set it equal to zero to solve for critical points.

Take the second derivative of the function.

Substitute the critical point in the second derivative.

If this value is negative, the critical point is a relative maximum.

If this value is positive, the critical point is a relative minimum.

If this value is zero, the critical point is a possible point of inflection. Test points on either side of the critical point by substituting them into the second derivative to verify that the concavity changed.

- **Mean Value Theorem**

If $f(x)$ is defined and continuous on $a \le x \le b$ and differentiable on $a < x < b$,

Then there is at least one number c between a and b where $f(b) - f(a) = f'(c)\left(b - a\right)$

In other words, $f'(c) = \dfrac{f(b) - f(a)}{b - a}$

Geometrically, there must be at least one point in the interval where the tangent to the curve at that point is parallel to the secant line which passes through the points (a, f(a)) and (b, f(b)).

- **exponential Growth and Decay**

$\dfrac{dy}{dt} = k\,y$ where k is called the constant of proportionality.

$y = N e^{kt}$ or $A = P e^{rt}$ or $y = C e^{kt}$

- **Newton's Method**
- **Implicit Differentiation**
- **Related Rates**
- **Applied Maxima / Minima**

Slope Fields