- **Integration by Trig Substitution**:

  If the integral contains trig expressions, try substituting some of the basic trig identities:

$$\sin\theta = \frac{1}{\csc\theta} \qquad \cos\theta = \frac{1}{\sec\theta} \qquad \tan\theta = \frac{1}{\cot\theta} \qquad \tan\theta = \frac{\sin\theta}{\cos\theta}$$

$$\cot\theta = \frac{1}{\tan\theta} \qquad \sec\theta = \frac{1}{\cos\theta} \qquad \csc\theta = \frac{1}{\sin\theta} \qquad \cot\theta = \frac{\cos\theta}{\sin\theta}$$

$$\cos^2\theta = \frac{1+\cos 2\theta}{2} \qquad \sin^2\theta = \frac{1-\cos 2\theta}{2}$$

$$\sin^2\theta + \cos^2\theta = 1 \qquad \tan^2\theta + 1 = \sec^2\theta \qquad 1 + \cot^2\theta = \csc^2\theta$$

$$\sin 2A = 2\sin A\cos A \qquad \cos 2A = \cos^2 A - \sin^2 A$$

  If the integral contains the sum or difference of two squares, set up right triangles and make appropriate trig substitutions.

- **Numerical Integration**: When symbolic methods fail, use of some numerical approximation method will give useful answers along a specified interval. Most calculators enact these methods to give extremely exact answers by using very tiny subdivisions.

  - **Riemann Sums with Rectangles:** Rectangular Approximation Method.

$$\int_a^b f(x)\,dx \approx \sum_{i=1}^{n}\left(f(x_i)\,\Delta x\right)$$

$$\approx \left(\frac{b-a}{n}\right)\left(y_0 + y_1 + y_2 + \ldots + y_{n-1}\right) \quad \{\text{Left-Hand Endpoints}\}$$

$$\approx \left(\frac{b-a}{n}\right)\left(y_1 + y_2 + y_3 + \ldots + y_n\right) \quad \{\text{Right-Hand Endpoints}\}$$

  - **Trapezoidal Rule:** Trapezoidal approximation.

$$\int_a^b f(x)\,dx \approx \left(\frac{b-a}{2n}\right)\left(y_0 + 2y_2 + 2y_3 + 2y_4 \ldots + 2y_{n-1} + y_n\right)$$