Applications of Integration

- **Rectilinear Motion**

Derivatives:

If $s = f(t)$

Then $v = \frac{ds}{dt} = s' = f'(t)$

Then $a = \frac{dv}{dt} = \frac{d^2s}{dt^2} = s'' = f''(t) = v'$

Integrals:

If $a = f(t)$

Then $v = \int a\, dt$

Then $s = \int v\, dt$

Recall the difference between Displacement and Total Distance.

- **Average Value of a Function**

$$\bar{Y} = \frac{1}{b-a}\int_a^b f(x)\,dx$$

- **Area Between Two Curves**

(1) If one function is always on top of the other function, integrate with respect to x:

$$A = \int_a^b \left(f(x) - g(x) \right) dx$$

(2) If one relation is always to the right of the other relation, integrate with respect to y:

$$A = \int_c^d \left(f(y) - g(y) \right) dy$$

- **Volumes of Solids of Revolution**

(1) Disk Method $V = \pi r^2 h$

About the x-axis: $V = \int_{x=a}^{b} \pi\left(f(x)\right)^2 dx$

About the y-axis: $V = \int_{y=c}^{d} \pi\left(f(y)\right)^2 dy$

(2) Washer Method $V = \pi R^2 h - \pi r^2 h$

About the x-axis: $V = \int_{x=a}^{b} \pi\left(f(x)\right)^2 dx - \int_{x=a}^{b} \pi\left(g(x)\right)^2 dx$

About the y-axis: $V = \int_{y=c}^{d} \pi\left(f(y)\right)^2 dy - \int_{y=c}^{d} \pi\left(g(y)\right)^2 dy$

(3) Shell Method $V = 2\pi r\, h(\text{thickness})$

About the x-axis: $V = \int_{y=c}^{d} 2\pi\, y\left(f(y)\right) dy$

About the y-axis: $V = \int_{x=a}^{b} 2\pi\, x\left(f(x)\right) dx$

- **Volumes of Cross-Sections**

$V = \int_{a}^{b} A(x)\,dx$ where $A(x)$ is the area of the cross section

- **Length of an Arc**

$L = \int_{x=a}^{b} \sqrt{1+\left(\frac{dy}{dx}\right)^2}\, dx$ $L = \int_{y=c}^{d} \sqrt{1+\left(\frac{dx}{dy}\right)^2}\, dy$ $L = \int_{t=e}^{f} \sqrt{\left(\frac{dx}{dt}\right)^2+\left(\frac{dy}{dt}\right)^2}\, dt$