Curve Sketching

Instructions for Curve Sketching… Given a function $y = f(x)$ to sketch,

1. Calculate $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$.

2. Set $\frac{dy}{dx} = 0$ and solve for x to find critical points.
   These points may be relative maximum points, relative minimum points, or points of inflection.

3. Substitute the values of x found in step 2 into the original function $y = f(x)$, and plot these coordinates.

4. Substitute the values of x found in step 2 into the formula for the second derivative $\frac{d^2y}{dx^2}$.

   A. If $\frac{d^2y}{dx^2} = 0$, Then it ***may*** be a point of inflection. You must test points on each side.

   Substitute a point before each value of x and a point after each value of x into the formula for the second derivative.
   If the values of the second derivative have opposite signs, then x is a point of inflection.

   B. If $\frac{d^2y}{dx^2} > 0$, Then x is a relative minimum point.

   C. If $\frac{d^2y}{dx^2} < 0$, Then x is a relative maximum point.

5. Remember that points of inflection (where concavity changes from positive to negative) may also occur where $\frac{d^2y}{dx^2}$ is undefined (a vertical tangent) or where $\frac{d^2y}{dx^2} = 0$.

6. Look for points of discontinuity.

7. Find values of x for which $\frac{dy}{dx}$ is positive and also for which $\frac{dy}{dx}$ is negative.

   Calculate the values of y and $\frac{d^2y}{dx^2}$ at points of transition between positive and negative values.

8. To locate relative minimum and relative maximum points, you can use the first derivative test instead of the second derivative test in #4 above.

Set $\frac{dy}{dx} = 0$ and solve for *x*. Then take points on either side of the value of *x* and substitute them into the derivative.

A. For $a < x < b$, if $f'(a) > 0$ and $f'(b) < 0$, then *x* is a relative maximum point.

B. For $a < x < b$, if $f'(a) < 0$ and $f'(b) > 0$, then *x* is a relative minimum point.

C. For $a < x < b$, if $f'(a) > 0$ and $f'(b) > 0$, then *x* is a point of inflection.

D. For $a < x < b$, if $f'(a) < 0$ and $f'(b) < 0$, then *x* is a point of inflection.

9. To check where a function is increasing / decreasing:

A. If $f'(x) > 0$, Then $f(x)$ is increasing.

B. If $f'(x) < 0$, Then $f(x)$ is decreasing.

10. To check where a function is concave up or down:

A. If $f''(x) > 0$, Then $f(x)$ is concave up at that point.

B. If $f''(x) < 0$, Then $f(x)$ is concave down at that point.

C. If $f''(x) = 0$, Then check points on either side to see if it is a point of inflection. Concavity must change in order for it to be a point of inflection.