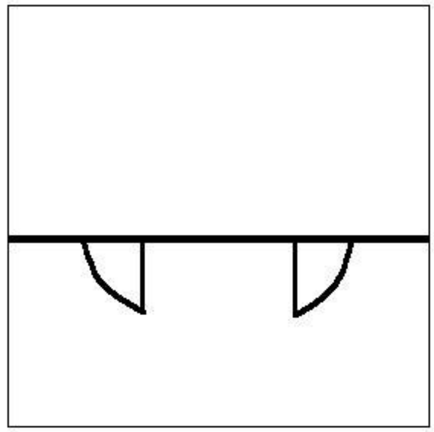
# Turvy for Related Rates – Answer Key



Here is the title right-side-up:

"A C L O S E - U P O F D R A C U L A."
5 2 6 3 13 4 7 5 2

Here is the title upside-down:

" T W O S H A R K S N O T O N S P E A K I N G
8 12 6 11 7 9 6 10 8 10 6 3 9 10 1

T E R M S."
8 7 14 6

To determine the titles to this turvy, solve the 14 Related Rates problems. Then replace each numbered blank with the letter corresponding to the answer for that problem. The unnumbered blanks are all vowels. You should be able to determine both titles.

1-2. A certain calculus student hit Mr. Pleacher in the head with a snowball. If the snowball is melting at the rate of 10 cubic feet per minute, at what rate is the radius changing when the snowball is 1 foot in radius (Problem #1)? At what rate is the radius changing when the snowball is 2 feet in radius (Problem #2)?
Answers should be expressed in terms of feet per minute.

$$V = \frac{4}{3}\pi r^3$$

$$\frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt}$$

$$-10 = 4\pi\left(1^2\right)\frac{dr}{dt}$$

$$\frac{dr}{dt} = \frac{-5}{2\pi}\ ft/\min \quad \therefore \#1 \quad G$$

$$-10 = 4\pi\left(2^2\right)\frac{dr}{dt}$$

$$\frac{dr}{dt} = \frac{-10}{16\pi} = \frac{-5}{8\pi}\ ft/\min \quad \therefore \#2\ L$$

3-4. A baseball diamond is 90 feet square (NOT 90 square feet!). Coach Jack Handley runs from first base to second base at 25 feet per second. How fast is he moving away from home plate when he is 30 feet from first base (Problem #3)? How fast is he moving away from home plate when he is 45 feet from first base (Problem #4)?
Answers should be expressed in terms of feet per second.

$$x^2 + 90^2 = y^2$$

$$2x\frac{dx}{dt} = 2y\frac{dy}{dt}$$

$$(30)(25) = \sqrt{900}\,\frac{dy}{dt}$$

$$\frac{dy}{dt} = \frac{5\sqrt{10}}{2}\ ft/s \quad \#3\ P$$

$$(45)(25) = \sqrt{10125}\,\frac{dy}{dt}$$

$$\frac{dy}{dt} = 5\sqrt{5}\ ft/s \quad \#4\ D$$

5. Water flows at 8 cubic feet per minute into a cylinder with radius 4 feet. How fast is the water level rising when the water is 2 feet high?
Answer should be expressed in terms of feet per minute.

$$V = \pi(4)^2 h$$

$$\frac{dV}{dt} = 16\pi\frac{dh}{dt}$$

$$8 = 16\pi\frac{dh}{dt}$$

$$\frac{dh}{dt} = \frac{1}{2\pi}\ ft/\min \qquad \#5\ C$$

6-7. The Monticello High School swimming pool is an inverted cone with height 20 meters and radius 5 meters. It is being filled by Mr. Blundin with a hose which pumps in water at the rate of 3 cubic meters per minute.
When the water level is 2 meters, how fast is the water level rising (Problem #6)?
How fast is the radius changing at this moment (Problem #7)?
Answers should be expressed in terms of meters per minute.

$$V = \frac{1}{3}\pi r^2 h \qquad \frac{r}{h} = \frac{5}{20} = \frac{1}{4} \qquad r = \frac{h}{4}$$

$$V = \frac{1}{3}\pi\left(\frac{h^2}{16}\right)h$$

$$V = \frac{1}{48}\pi h^3$$

$$\frac{dV}{dt} = \frac{1}{16}\pi h^2 \frac{dh}{dt}$$

$$3 = \frac{1}{16}\pi 2^2 \frac{dh}{dt}$$

$$\frac{dh}{dt} = \frac{12}{\pi}\ m/\min \qquad \#6\ S$$

$$h = 4r$$

$$V = \frac{1}{3}\pi r^2 (4r)$$

$$V = \frac{4}{3}\pi r^3 \qquad \frac{r}{h} = \frac{1}{4}$$

$$\frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt} \qquad \frac{r}{2} = \frac{1}{4}$$

$$3 = 4\pi \frac{1}{4}\frac{dr}{dt} \qquad r = \frac{1}{2}$$

$$\frac{dr}{dt} = \frac{3}{\pi}\ m/\min \qquad \#7\ R$$

8-9. A stone is dropped into Sherando Lake, causing circular ripples whose radii increase by 2 meters/second. How fast is the disturbed area growing when the outer ripple has radius 5 meters (Problem #8)? How fast is the radius increasing at that moment (Problem #9)?
Answers should be expressed in terms of square meters per second (#8) and meters per second (#9).

$$A = \pi r^2$$

$$\frac{dA}{dt} = 2\pi r \frac{dr}{dt}$$

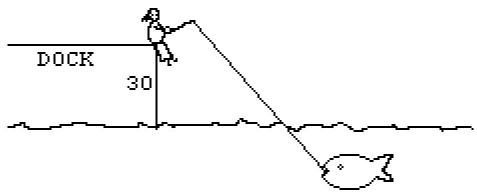
$$\frac{dA}{dt} = 2\pi(5)(2)$$

$$\frac{dA}{dt} = 20\pi\ \ m^2/s \qquad \#8\ T$$

$$\frac{dr}{dt} = 2\ m/s \qquad \#9\ K$$

10-11. A fish is being reeled in at a rate of 2 meters / second (that is, the fishing line is being shortened by 2 m/s) by a fisherwoman at Mill brook. If the fisherwoman is sitting on the dock 30 meters above the water, how fast is the fish moving through the water when the line is 50 meters long (Problem #10)? How fast is the fish moving when the line is only 31 meters (Problem #11)?

Answers should be expressed in terms of meters per second.




$$30^2 + x^2 = z^2$$

$$2x\frac{dx}{dt} = 2z\frac{dz}{dt}$$

$$40\frac{dx}{dt} = (50)(2)$$

$$\frac{dx}{dt} = \frac{5}{2}\ ft/s \qquad \#10\ N$$

$$\sqrt{61}\frac{dx}{dt} = 31(2)$$

$$\frac{dx}{dt} = \frac{62}{\sqrt{61}}\quad ft/s \qquad \#11\ H$$

12. A student at James Wood was painting the high school and standing at the top of a 25-foot ladder. She was horrified to discover that the ladder began sliding away from the base of the school at a constant rate of 2 feet per second. At what rate was the top of the ladder carrying her toward the ground when the base of the ladder was 17 feet away from the school?

Answers should be expressed in terms of feet per second.

$$x^2 + y^2 = 25^2$$

$$2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 0$$

$$(17)(2) + \sqrt{336}\frac{dy}{dt} = 0$$

$$\frac{dy}{dt} = \frac{-34}{\sqrt{336}} = -1.855\ ft/s \qquad \#12\ W$$

13. A spherical balloon was losing air at the rate of 5 cubic inches per second. At what rate is the radius of the balloon decreasing when the radius equals 5 inches? Answers should be expressed in terms of inches per second.

$$V = \frac{4}{3}\pi r^3$$
$$\frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt}$$
$$-5 = 4\pi(25)\frac{dr}{dt}$$
$$\frac{dr}{dt} = \frac{-1}{20\pi}\ in/s \qquad \#13\ F$$

14. Oil spills into Lake Winchester in a circular pattern. If the radius of the circle increases at a constant rate of 3 feet per minute, how fast is the area of the spill increasing at the end of 10 minutes? Answers should be expressed in terms of feet per minute.

$$A = \pi r^2$$
$$\frac{dA}{dt} = 2\pi r \frac{dr}{dt}$$
$$\frac{dA}{dt} = 2\pi(3x30)(3)$$
$$\frac{dA}{dt} = 180\pi\ ft^2/\min \qquad \#14\ M$$

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Answers:

| B | C | D | F | G | H | K | L |
|---|---|---|---|---|---|---|---|
| $O$ | $\frac{1}{2\pi}$ | $5\sqrt{5}$ | $-\frac{1}{20\pi}$ | $-\frac{5}{2\pi}$ | $\frac{62}{61}\sqrt{61}$ | 2 | $-\frac{5}{8\pi}$ |

| M | N | P | R | S | T | V | W |
|---|---|---|---|---|---|---|---|
| $180\pi$ | $\frac{5}{2}$ | $\frac{5\sqrt{10}}{2}$ | $\frac{3}{\pi}$ | $\frac{12}{\pi}$ | $20\pi$ | $\frac{-17\sqrt{336}}{336}$ | -1.855 |