Calculus for all seasons
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It seems that when we teach functions we always have a particular function defined by a "formula" on the chalkboard. When we talk about where a function is increasing or decreasing, we always have a given function before us. And heaven forbid that we discuss derivatives without calculating one. The best way around these difficulties is to choose a simple example from the "real world" and attempt to model it.

We can start by asking, "What are the seasons?" We can lead our students to the following (nonstandard) definition of the seasons.

"Winter is when, in a given 24-hour period, the dark time is longer than the light time and each light time is longer than the last one.

Spring is when the light time is longer than the dark time and each light time is longer than the last one.

Summer is when the light time is longer than the dark time and each light time is shorter than the last one.

Finally, Autumn is when the light time is shorter than the dark time and each light time is shorter than the last one."

Once we arrive at this formulation for the seasons, it becomes clear that we should be able to describe the seasons completely in terms of the amount of light time in a given one day (4-hour) period. We shall do so with a function.

We let t be the time, in days, since some arbitrary starting point occurred. Then our function f will give the total light time between one-half day before t and one-half day after t less one-half day. If we insist on a formula,
f(t) =total time of light in days during [t-1/2, t+1/2] – 1/2 day.

It is then easy to characterize the seasons:

	$f \geq 0$	$f < 0$
f increasing or $f' \geq 0$	Spring	Winter
f increasing or $f' < 0$	Summer	Autumn

We have a "natural function" for which it makes sense to discuss zeros, increase, decrease, and critical values. The zero between summer and autumn is approximately the autumnal equinox, and the other zero is approximately the vernal equinox. The critical value between spring and summer approximately the summer solstice, and the other is approximately the winter solstice.

Finally, we observe that f is periodic with fundamental period 365.