Sudoku Puzzle – Limits and Derivatives for A.P. Calculus
A Puzzle by David Pleacher

Solve the 30 multiple-choice problems below.
The choices are integers from 1 to 9 inclusive.
Place the answer in the corresponding cell (labeled A, B, C, … Y, Z, a, b,c,d).
Then solve the resulting SUDOKU puzzle.

The rules of Sudoku are simple.
Enter digits from 1 to 9 into the blank spaces.
Every row must contain one of each digit.
So must every column, and so must every 3x3 square.
Each Sudoku has a unique solution that can be reached logically without guessing.

_____ A. A bouncing ball **loses** ¼ of its previous height each time that it rebounds. If the ball is thrown up to a height of 60 feet, how many feet will it travel before coming to rest?

(5) 240 (6) 480 (7) 160 (8) 80 (9) 120

_____ B. If $f(x) = x + \sin(x)$, then $f'(x) =$

(5) $1 - \cos(x)$ (6) $\cos(x)$ (7) $\sin(x) - x\cos(x)$
(8) $\sin(x) + x\cos(x)$ (9) $1 + \cos(x)$

_____ C. If $f(x) = \dfrac{x-1}{x+1}$ for all $x \neq -1$, then $f'(1) =$

(5) -1 (6) $-\dfrac{1}{2}$ (7) 0 (8) $\dfrac{1}{2}$ (9) 1

_____ D. If $y = \cos^2 3x$, then $\dfrac{dy}{dx} =$

(5) $-2\cos 3x$ (6) $-6\sin 3x\cos 3x$ (7) $2\cos 3x$
(8) $6\cos 3x$ (9) $2\sin 3x\cos 3x$

_____ E. The derivative of $f(x) = \dfrac{x^4}{3} - \dfrac{x^5}{5}$ attains its maximum value at $x =$

(1) -1 (2) 0 (3) $\dfrac{4}{5}$ (4) 1 (5) $\dfrac{5}{3}$

______ F. If the line $3x-4y=0$ is tangent in the first quadrant to the curve $y=x^3+k$, then $k=$

(1) $\frac{1}{2}$ (2) $\frac{1}{4}$ (3) 0 (4) $-\frac{1}{8}$ (5) $-\frac{1}{2}$

______ G. If $f(x)=(x)^{\frac{1}{3}}(x-2)^{\frac{2}{3}}$, then the domain of $f'(x)$ is:

(1) $\{x \mid x \neq 0\}$

(2) $\{x \mid x \neq 0 \text{ and } x \neq 2\}$

(3) $\{x \mid x > 0\}$

(4) $\{x \mid 0 \leq x \leq 2\}$

(5) $\{x \mid x \text{ is a real number}\}$

______ H. If $\frac{d}{dx}(f(x))=g(x)$ and $\frac{d}{dx}(g(x))=f(x^2)$,

Then $\frac{d^2}{dx^2}(f(x^3))=$

(1) $9x^4 f(x^6)+6x\,g(x^3)$ (2) $f(x^6)+g(x^3)$

(3) $f(x^6)$ (4) $g(x^3)$ (5) $3x^2 g(x^3)$

______ I. Which of the following defines a function f for which $f(-x)=-f(x)$?

(6) $f(x)=x^2$ (7) $f(x)=\sin(x)$ (8) $f(x)=\cos(x)$

______ J. Determine $\lim_{h\to 0}\frac{8\left(\frac{1}{2}+h\right)^8-8\left(\frac{1}{2}\right)^8}{h}$

(6) 0 (7) 1 (8) $\frac{1}{2}$ (9) The limit does not exist

______ K. Determine $\lim_{x\to 0}\frac{\sin(10x)}{x}$

(1) 0 (2) .1 (3) 10 (4) The limit does not exist

_____ L. Determine $\lim_{p\to 0}\frac{(x+p)^n - x^n}{p}$

(1) np (2) nx^{n-1} (3) np^{n-1} (4) 0

(5) The limit can not be determined

_____ M. Determine $\lim_{\theta\to 0}\left(\frac{\sin\theta\tan\theta}{1-\cos\theta}\right)$

(4) 0 (5) 1 (6) 2 (7) The limit does not exist

_____ N. Determine $g'\left(\frac{\pi}{6}\right)$ if $g(x)=\sin^3(2x)$

(5) 4.5 (6) 0 (7) 9 (8) 1 (9) 2.25

_____ O. Determine the value of k if the graphs of $y=-2x$ and $y=\sin k\pi x$ are tangent at $(0,0)$.

(5) -2 (6) π (7) $\frac{\pi}{2}$ (8) $\frac{-\pi}{2}$ (9) $\frac{-2}{\pi}$

_____ P. If $f(x)=x^2+g^3(x)$, Determine $f'(3)$ if $g(3)=2$ and $g'(3)=-1$.

(1) -6 (2) -1 (3) 0 (4) 12 (5) 18

_____ Q. Given $y^3+y=x$, Determine the value of y' at $(2,1)$.

(1) 0 (2) .5 (3) .25 (4) 1 (5) 10

_____ R. If $y=(ax+1)^{-2}$, determine the 10^{th} derivative of y with respect to x.

(1) $-2^{10}(ax+1)^{-12}$ (2) $\frac{-11!a^{10}}{(ax+1)^{12}}$ (3) $\frac{11!}{(ax+1)^{12}}$ (4) $\frac{11!a^{10}}{(ax+1)^{12}}$

_____ S. If $y=(ax+1)^{-2}$, determine the 100^{th} derivative of y with respect to x.

(6) $-2^{100}(ax+1)^{-102}$ (7) $\frac{101!a^{100}}{(ax+1)^{102}}$ (8) $\frac{-101!a^{100}}{(ax+1)^{102}}$ (9) $\frac{101!}{(ax+1)^{102}}$

_____ T. If $y=(ax+1)^{-2}$, determine the n^{th} derivative of y with respect to x.

(6) $\frac{(-1)^n(n+1)!a^n}{(ax+1)^{(n+2)}}$ (7) $\frac{(-1)^n(n+1)!}{(ax+1)^{(n+2)}}$

(8) $\frac{-(n+1)!a^n}{(ax+1)^{(n+2)}}$ (9) $\frac{(n+1)!a^n}{(ax+1)^{(n+2)}}$

______ U. Determine the derivative with respect to x of $\frac{\cos x}{x\sin x}$.

(3) $\frac{-x\sin^2 x-\cos x\sin x}{x^2\sin^2 x}$ (4) $\frac{\cos x\sin x+x}{x^2\sin^2 x}$

(5) $\frac{-x-\cos x\sin x}{x^2\sin^2 x}$ (6) $\frac{x-\cos x\sin x}{x^2\sin^2 x}$

______ V. Given $x^3+y^3=9,$ Determine the value of y'' at $(2,1)$.

(3) 32 (4) 24 (5) 0 (6) -36 (7) -48

______ W. Determine $g'(4)$ if $g(x)=\left(1-\sqrt{x}\right)^4$

(6) 0 (7) 1 (8) 4 (9) -1

______ X. Determine $\lim_{h\to 0}\frac{\sqrt{8+h}-\sqrt{8}}{h}$

(1) 0 (2) $\frac{\sqrt{2}}{8}$ (3) $\frac{\sqrt{2}}{4}$ (4) The limit does not exist

______ Y. Determine the differential of $y=\cos\left(\sin\left(x^2+3x\right)\right)$

(1) $dy=\left(-\sin\left(\sin\left(x^2+3x\right)\right)\cos\left(x^2+3x\right)(2x+3)\right)\text{dx}$

(2) $dy=\left(-\sin\left(\sin\left(x^2+3x\right)\right)(2x+3)\right)\text{dx}$

(3) $dy=\left(-\sin\left(\cos\left(x^2+3x\right)\right)(2x+3)\right)\text{dx}$

(4) none of the above

______ Z. Determine $\lim_{x\to 0}\frac{\sin(4x)}{x}$

(4) 0 (5) 4 (6) $\frac{1}{4}$ (7) The limit does not exist

______ a. If $x=t^2$ and $y=2t-3,$ then $\frac{dy}{dx}=$

(2) $\left(\frac{y+3}{2}\right)^2$ (3) $\frac{2}{y+3}$ (4) $\frac{y+3}{4}$ (5) $2\sqrt{x}-3$

_____ b. Determine the 50th derivative of y with respect to x of $y=(5x+1)^{-2}$

(2) $\dfrac{51!}{(5x+1)^{52}}$ (3) $\dfrac{5^{50}\bullet 51!}{(5x+1)^{51}}$ (4) $\dfrac{5^{50}\bullet 51!}{(5x+1)^{52}}$ (5) $\dfrac{51!}{(5x+1)^{50}}$

_____ c. Determine the equation of the line that is normal to the curve $x^2+y=5$ at the point $(2,1)$.

(6) $y-1=-4(x-2)$ (7) $y-1=-2x(x-2)$

(8) $y-1=\dfrac{-1}{4}(x-2)$ (9) $y-1=\dfrac{1}{4}(x-2)$

_____ d. A "bug" of negligibile dimensions starts at the origin $(0,0)$ of the standard two-dimensional rectangular coordinate system. The bug walks one unit to the right, then one-half unit up, then $\dfrac{1}{4}$ left, then $\dfrac{1}{8}$ down, etc.

In each move, it always turns counter-clockwise at a 90° angle and goes half the distance it went on the previous move.

Which point (x,y) in the xy-plane is the bug approaching in its spiraling journey?

(5) $\left(\dfrac{4}{5},\dfrac{2}{5}\right)$ (6) $\left(\dfrac{4}{5},\dfrac{1}{5}\right)$ (7) $(1,1)$

(8) $(2,1)$ (9) $\left(\dfrac{1}{2},\dfrac{1}{2}\right)$

A	B							C
				D	E			F
		G		H			I	
J			K	L	M			N
O	P					Q		
		R			S		T	
		U	V	W		X		
					Y	Z	a	
b		c		d				

Here is a blank SUDOKU board for you to use:

Solution to the Sudoku With Limits and Derivatives

A = 6
B = 9
C = 8
D = 6

E = 4
F = 2
G = 2
H = 1

I = 7
J = 8
K = 3
L = 2

M = 6
N = 9
O = 9
P = 1

Q = 3
R = 4
S = 7
T = 6

U = 5
V = 6
W = 7
X = 2

Y = 1
Z = 5
a = 3
b =4

c = 9
d = 5

6	9	1	7	3	2	4	5	8
7	8	3	5	6	4	9	1	2
5	4	2	8	1	9	6	7	3
8	5	7	3	2	6	1	4	9
9	1	6	4	8	5	3	2	7
3	2	4	1	9	7	8	6	5
1	3	5	6	7	8	2	9	4
2	7	8	9	4	1	5	3	6
4	6	9	2	5	3	7	8	1

Many thanks to Emanuel Dicker for correcting an error in the puzzle.