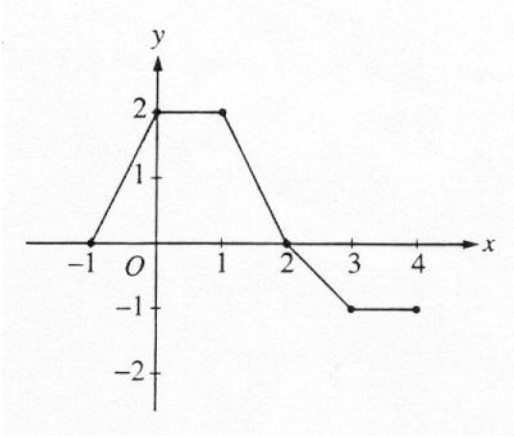
Sudoku Puzzle – A.P. Exam (Part A)
From the 1998 A.P. Exam
A Puzzle by David Pleacher

Solve the 26 multiple-choice problems below.
A calculator is not allowed for any of these questions.
The choices are integers from 1 to 9 inclusive.
Place the answer in the corresponding cell (labeled A, B, C, … Y, Z).
Then solve the resulting SUDOKU puzzle.

The rules of Sudoku are simple.
Enter digits from 1 to 9 into the blank spaces.
Every row must contain one of each digit.
So must every column, and so must every 3x3 square.
Each Sudoku has a unique solution that can be reached logically without guessing.

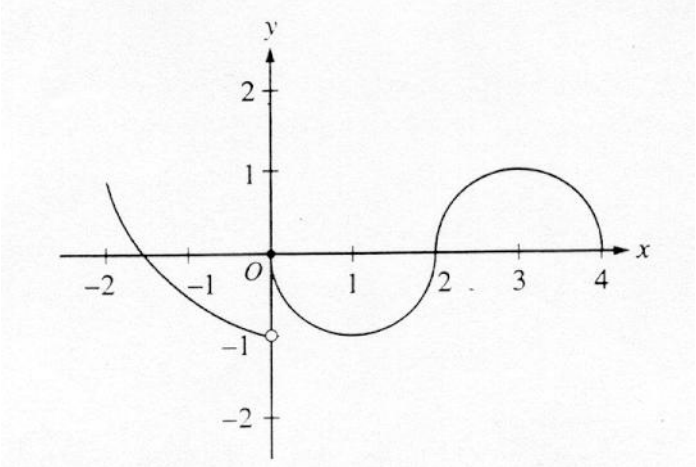
_____ A. What is the x-coordinate of the point of inflection on the graph of $y = \frac{1}{3}x^3 + 5x^2 + 24$?

(1) -10 (2) 0 (3) -5 (4) $\frac{-10}{3}$ (5) 5



_____ B. The graph of a piecewise-linear function f, for $-1 \le x \le 4$, is shown above. What is the value of $\int_{-1}^{4} f(x)\,dx$?

(5) 1 (6) 8 (7) 5.5 (8) 4 (9) 2.5



_____ C. The graph of the function f shown in the figure above has a vertical tangent at the point $(2,0)$ and horizontal tangents at the points $(1,-1)$ and $(3,1)$. For what values of x, $-2 < x < 4$, is f not differentiable?

(1) 0 only (2) 0 and 2 only (3) 1 and 3 only

(4) 0, 1, and 3 only (5) 0, 1, 2, and 3

_____ D. A particle moves along the x-axis so that its position at time t is given by $x(t) = t^2 - 6t + 5$. For what value of t is the velocity of the particle zero?

(5) 5 (6) 1 (7) 2 (8) 3 (9) 4

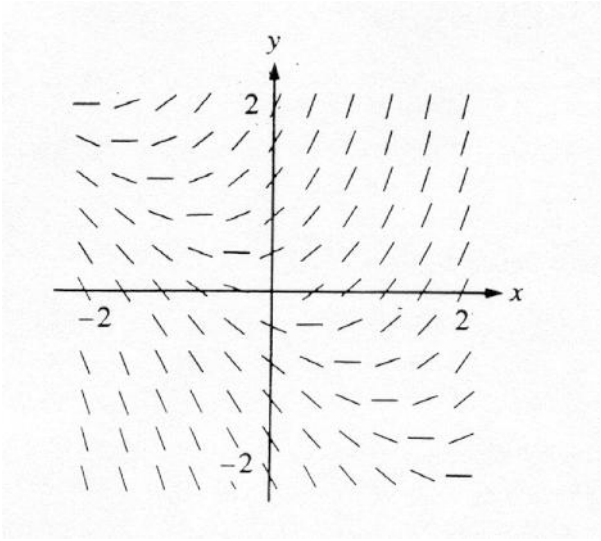
_____ E. If $F(x) = \int_0^x \sqrt{t^3+1}\,dt$ then $F'(2) =$

(1) 3 (2) 2 (3) -3 (4) -2 (5) 18

_____ F. If $f(x) = \sin(e^{-x})$, then $f'(x) =$

(4) $-\cos(e^{-x})$ (5) $\cos(e^{-x}) - e^{-x}$ (6) $\cos(e^{-x}) + e^{-x}$

(7) $-e^{-x}\cos(e^{-x})$ (8) $e^{-x}\cos(e^{-x})$



_____ G. Shown above is a slope field for which of the following differential equations?

(4) $\frac{dy}{dx} = 1 + x$ (5) $\frac{dy}{dx} = x^2$ (6) $\frac{dy}{dx} = \frac{x}{y}$

(7) $\frac{dy}{dx} = \ln y$ (8) $\frac{dy}{dx} = x + y$

_____ H. What is the area of the region between the graphs of $y = x^2$ and $y = -x$ from $x = 0$ to $x = 2$?

(1) $\frac{2}{3}$ (2) $\frac{8}{3}$ (3) 4 (4) $\frac{14}{3}$ (5) $\frac{16}{3}$

---

x	0	1	2
f(x)	1	k	2

_____ I. The function f is continuous on the closed interval [0, 2] and has values that are given in the table above. The equation $f(x) = \frac{1}{2}$ must have at least two solutions in the interval {0, 2} if $k =$

(5) 3 (6) 2 (7) 1 (8) $\frac{1}{2}$ (9) 0

_____ J. What is the average value of $y = x^2\sqrt{x^3+1}$ on the interval [0, 2]?

(1) $\frac{26}{9}$ (2) $\frac{52}{9}$ (3) $\frac{26}{3}$ (4) $\frac{52}{3}$ (5) 24

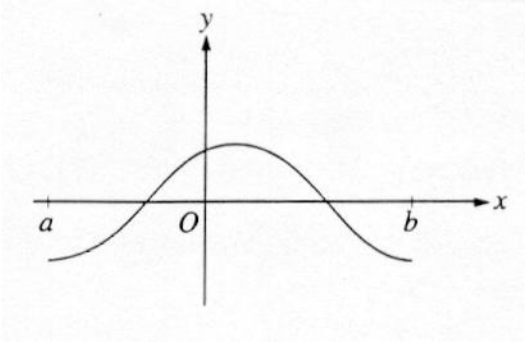
_____ K. If $f(x) = \tan(2x)$, then $f'\left(\frac{\pi}{6}\right) =$

(1) $4\sqrt{3}$ (2) 8 (3) 4 (4) $2\sqrt{3}$ (5) $\sqrt{3}$

_____ L. The function f is given by $f(x) = x^4 + x^2 - 2$.
On which of the following intervals is f increasing?

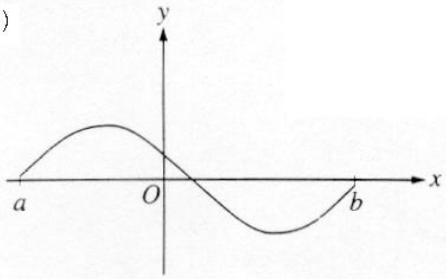
(1) $\left(-\frac{1}{\sqrt{2}}, \infty\right)$ (2) $\left(-\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$ (3) $(-\infty, 0)$

(4) $(0, \infty)$ (5) $\left(-\infty, -\frac{1}{\sqrt{2}}\right)$

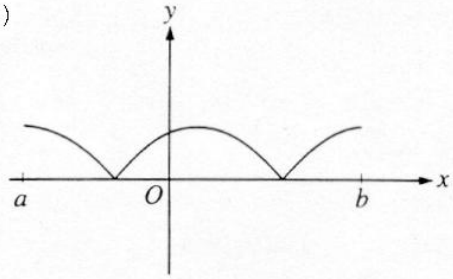


_____ M. The graph of f is shown in the figure above.
Which of the following could be the graph of the derivative of f?

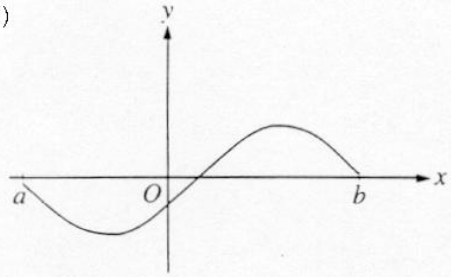
(3)



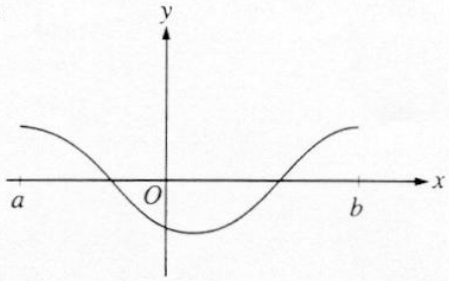
(4)



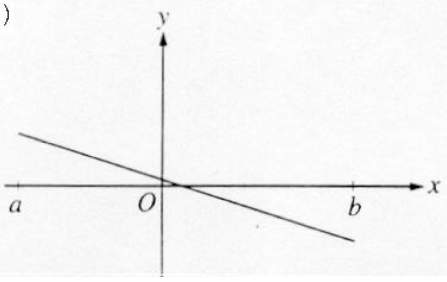
(5)

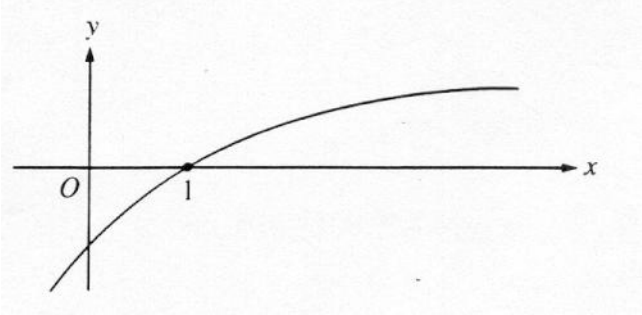


(6)



(7)





_____ N. The graph of a twice differentiable function f is shown above.
Which of the following is true?

(1) $f(1) < f'(1) < f''(1)$
(2) $f(1) < f''(1) < f'(1)$
(3) $f'(1) < f(1) < f''(1)$
(4) $f''(1) < f(1) < f'(1)$
(5) $f''(1) < f'(1) < f(1)$

_____ O. An equation of the line tangent to the graph of $y = x + \cos x$ at the point (0, 1) is

(1) $y = x + 1$ (2) $y = 2x + 1$ (3) $y = x$ (4) $y = x - 1$ (5) $y = 0$

_____ P. Determine the constant k if $x - 3$ is a linear factor of
$3x^3 - 9x^2 + kx - 12$.

(5) -4 only (6) 2 only (7) 4 only
(8) -4 and 3 only (9) $-1, 0,$ and 4 only

_____ Q. What are all the real values of k for which $\int_{-3}^{k} x^2\,dx = 0$?

(1) 0 (2) 3 (3) -3 (4) -3 and 3 (5) -3, 0 and 3

_____ R. If $\frac{dy}{dt} = ky$ and k is a nonzero constant, Then y could be

(5) $\frac{1}{2}ky^2 + \frac{1}{2}$ (6) $kty + 5$ (7) $e^{kt} + 3$ (8) $2e^{kty}$ (9) $2e^{kt}$

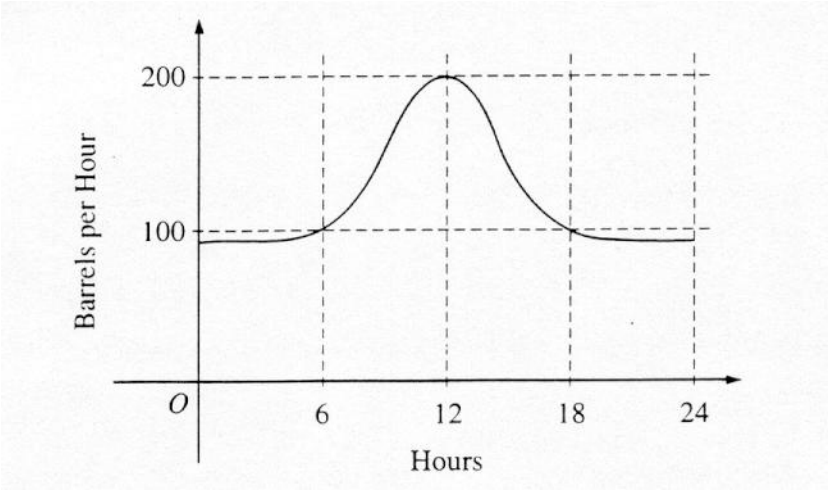
_____ S. Let f and g be differentiable functions with the following properties:

(i) $g(x) > 0$ for all x

(ii) $f(0) = 1$

If $h(x) = f(x)g(x)$ and $h'(x) = f(x)g'(x)$, then $f(x) =$

(5) $f'(x)$ (6) $g(x)$ (7) e^x (8) 1 (9) 0



_____ T. The flow of oil, in barrels per hour, through a pipeline on July 9 is given by the graph shown above. Of the following, which best approximates the total number of barrels of oil that passed through the pipeline that day?

(1) 500 (2) 600 (3) 3,000 (4) 4,800 (5) 2,400

_____ U. What is the instantaneous rate of change at $x = 2$ of the function f given by $f(x) = \dfrac{x^2 - 2}{x - 1}$?

(5) 6 (6) 2 (7) $\frac{1}{2}$ (8) $\frac{1}{6}$ (9) –2

___ V. If f is a linear function and $0 < a < b$, then $\int_a^b f''(x) =$

(4) 0 (5) 1 (6) $\frac{ab}{2}$ (7) $b-a$ (8) $\frac{b^2-a^2}{2}$

_____ W. If $f(x) = \begin{cases} \ln x & \text{for } 0 < x \le 2 \\ x^2 \ln 2 & \text{for } 2 < x \le 4, \end{cases}$ then $\lim_{x \to 2} f(x)$ is

(1) $\ln 2$ (2) $\ln 8$ (3) $\ln 16$ (4) 4 (5) nonexistent

____ X. $\int_1^2 \frac{1}{x^2} dx =$

(3) $-\frac{1}{2}$ (4) $\frac{7}{24}$ (5) $\frac{1}{2}$ (6) 1 (7) $2 \ln 2$

_____ Y. $\int_0^x \sin t \, dt =$

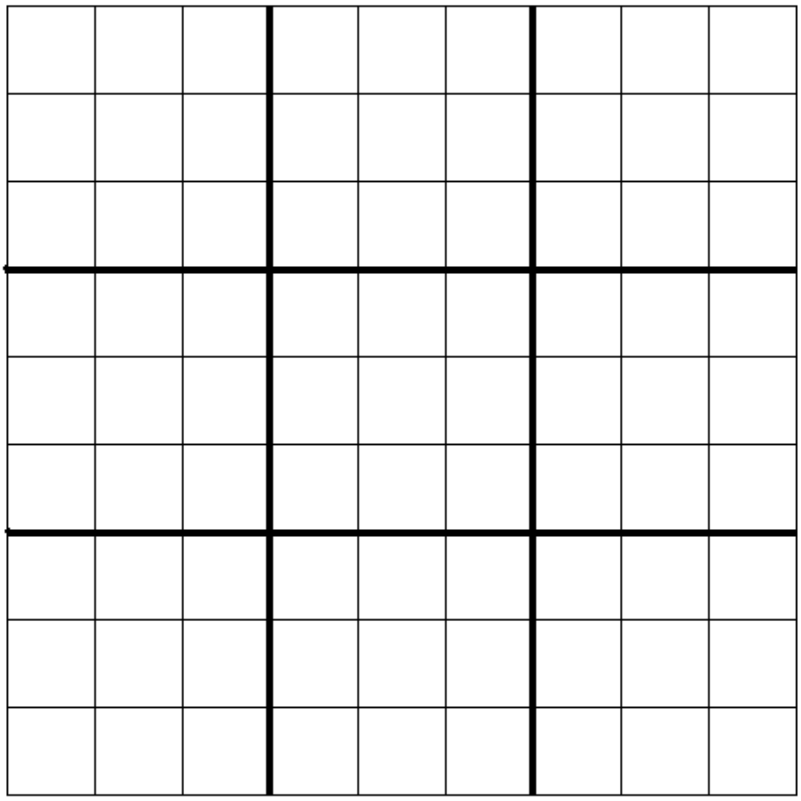
(3) $\sin x$ (4) $-\cos x$ (5) $\cos x$ (6) $\cos x - 1$ (7) $1 - \cos x$

_____ Z. $\int_1^e \left(\frac{x^2 - 1}{x} \right) dx =$

(3) $e - \frac{1}{e}$ (4) $e^2 - e$ (5) $e^2 - 2$ (6) $\frac{e^2}{2} - \frac{3}{2}$ (7) $\frac{e^2}{2} - e + \frac{1}{2}$

A			B		C	D		
E							F	
				G				H
					I		J	
		K	L		M			
N				O		P	Q	R
S								T
		U		V			W	
X			Y			Z		

Here is a blank SUDOKU board for you to use:



Solution to the Sudoku (A.P. Exam Part A)

A = 3
B = 9
C = 2
D = 8

E = 1
F = 7
G = 8
H = 4

I = 9
J = 1
K = 2
L = 4

M = 3
N = 4
O = 1
P = 7

Q = 3
R = 9
S = 8
T = 3

U = 6
V = 4
W = 5
X = 5

Y = 7
Z = 6

3	4	7	9	5	2	8	6	1
1	8	9	6	3	4	2	7	5
6	2	5	1	8	7	3	9	4
7	5	3	8	6	9	4	1	2
9	1	2	4	7	3	5	8	6
4	6	8	2	1	5	7	3	9
8	7	1	5	2	6	9	4	3
2	9	6	3	4	8	1	5	7
5	3	4	7	9	1	6	2	8