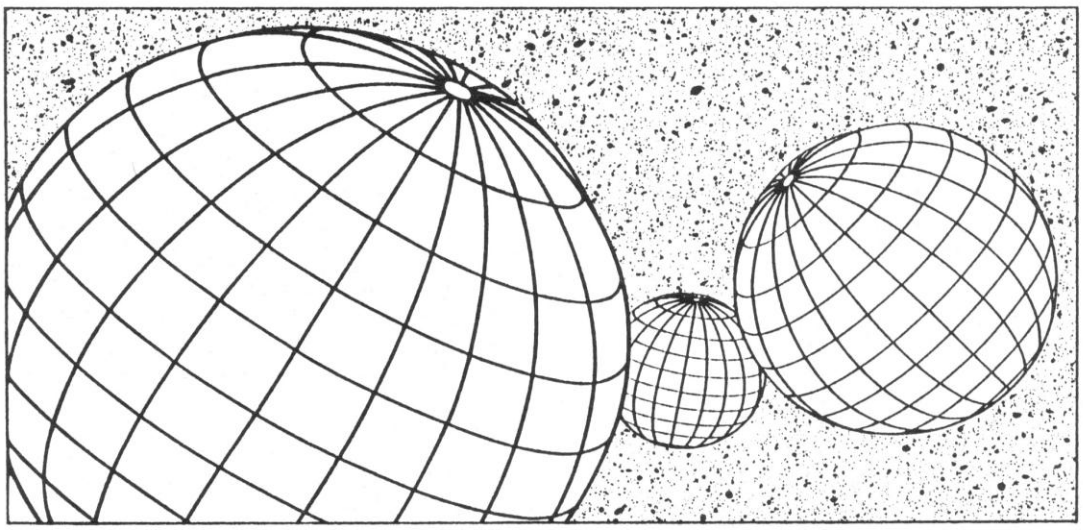
HiMAP Pull-Out Section: Summer 1991

For Earthlings, Eccentricity is 0.01673



Floyd Vest
Virginia Rawlins

References

Rudd, Warren L., and Terry Shell. 1990. *Prelude to Calculus*. Belmont, CA: Wadsworth Publishing Co.

Wilson, C. March 1972. How Did Kepler Discover His First Two Laws? *Scientific American*.

World Book Encyclopedia. See such topics as Solar System, Halley's Comet, Asteroid, etc.

As a citizen of Planet Earth, you can be called an Earthling. For the purpose of cosmic communications, you can be identified as a biological being of the third planet in orbit to an average G2-type star that Earthlings call the Sun. This average star and all members of its system, called the solar system, are members of the Milky Way Galaxy, which is merely one of the thousands of galaxies in your universe. Your planet, the planet Earth, can be identified by its orbit with the Sun at a focus. (See **Figure 1**.)

A solar system is composed of a star and planets which revolve around it. The star emits its own light, while the planets do not emit light but can reflect it. Objects which revolve around the planets are called satellites.

Kepler's first law of planetary motion states that all the planets and other orbiting bodies of our solar system travel around the Sun in elliptical orbits.

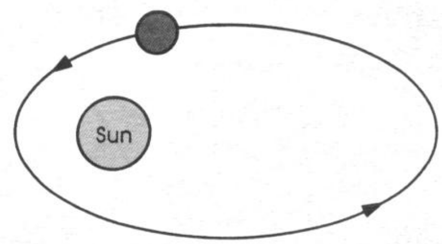


Figure 1:

Elliptical orbit of Earth with Sun at the focus.

These elliptical orbits are of very different shapes depending on the length of the semimajor axis and the eccentricity. Eccentricity is defined as a mathematical constant that for a given conic section is the ratio of the distances from any point of the conic section to a focus and the corresponding directrix. For an ellipse, eccentricity is a number greater than zero and less than one. As illustrated in **Figure 2**, the greater the eccentricity, the more elliptical the orbit.

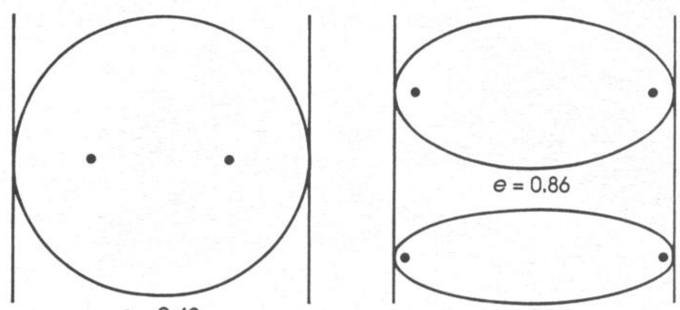


Figure 2:

Ellipses with different eccentricities. The two points within each ellipse are the foci.

The ellipse of the Earth's orbit is very close to circular: its eccentricity is 0.01673. On the other hand, the orbit of Halley's Comet is very eccentric ($e = 0.97214$). A chart of bodies in our solar system and information about their elliptical orbits is in **Figure 3**.

Halley's Comet is a brilliant comet named for the English astronomer Edmund Halley. It reappears about every 77 years.

Figure 3:

Solar system chart.

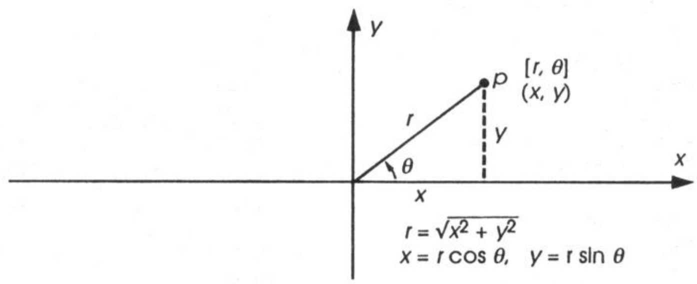
Body	Semimajor Axis of Orbit (in AU)	Eccentricity of Orbit
Mercury	0.3871	0.20563
Venus	0.7233	0.00679
Earth	1	0.01673
Mars	1.5237	0.09337
Jupiter	5.2028	0.07650
Saturn	9.5388	0.04844
Uranus	19.182	0.04721
Neptune	30.058	0.00858
Pluto	39.439	0.25024
Comet Halley	36.178	0.97214
Comet Kahotek	3466.7	0.99993
Asteroid Icarus	2.165	0.82712

An astronomical unit (AU) is a standard measurement of distance used in describing distances in our solar system. By definition, 1 AU is the length of the semimajor axis of the elliptical orbit of the earth. We will use 1 AU = 92,900,000 miles. (From looking in over 20 astronomy books, the value for one AU with the largest number of significant digits was 1 AU = 1.4959787 x 10⁸ km. Therefore, 1 AU = 92,975,681 miles (approximately).)

Equations of ellipses can be given in polar form where each point is identified by polar coordinates $[r, \theta]$ as in **Figure 4**. For point P with rectangular coordinates (x, y), we use $r = \sqrt{x^2 + y^2}$ and the angle θ as polar coordinates $[r, \theta]$.

Figure 4:

Polar coordinates.



Referring to **Figure 5**, we can define an ellipse (or any conic section) as a set of points satisfying the requirement that

$$\frac{\text{distance between the point and the focus}}{\text{distance between the point and the directrix}} = \frac{d_1}{d_2} = e \text{ (eccentricity)}.$$

The focus F is at the origin (pole of the coordinate system). The ellipse has a directrix (a reference line) whose rectangular equation is $x = -p$, where $p > 0$. Substituting $r \cos \theta = x$, we get $r \cos \theta = -p$ for the equation of the directrix, and thus $r = -p/\cos \theta$ and $r = -p \sec \theta$ for its equation given in Figure 5. Examining Figure 5 should reveal that $d_1 = r$ and $d_2 = p + r \cos \theta$ so that,

$$\frac{d_1}{d_2} = \frac{r}{p + r \cos \theta} = e.$$

Using algebra to rearrange the equation gives,

$$r = \frac{ep}{1 - e \cos \theta}$$ as the polar equation for an ellipse.

Figure 5:

Ellipse with focus at the pole.

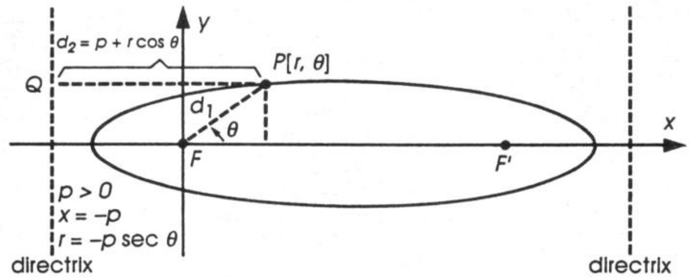
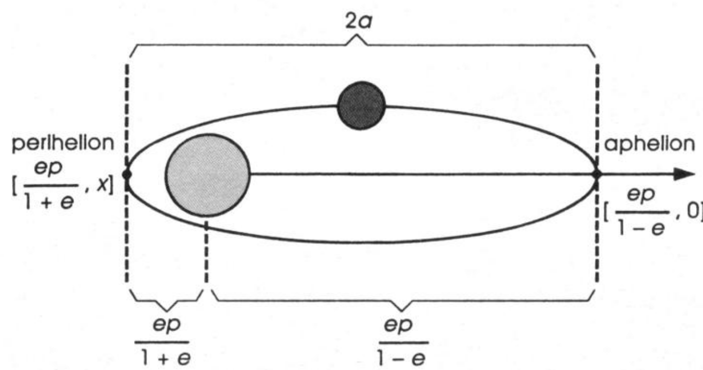


Figure 6:

Ellipse with major axis, perihelion, and aphelion.



Example 1:

Use the solar system chart in **Figure 3** to find the equation of the orbit of the planet Pluto.

For Pluto, $e = 0.25024$. (Pluto's orbit is the most elliptical of the planets.) The length of its semimajor axis is 39.439 AU. Substituting into the above equation, we have

$$r = \frac{39.439(1 - 0.25024^2)}{1 - 0.25024\cos\theta}$$ for the

equation of the orbit of Pluto.

Example 2:

Find the distance of the planet Pluto from the sun at the perihelion and at the aphelion.

Refer to **Figure 6** and to the equation of the planet Pluto in **Example 1**. To find the perihelion, let $\theta = \pi$ and calculate

$$r = \frac{39.439(1 - 0.25024^2)}{1 - 0.25024(-1)} = 29.570 \text{ AU}.$$

For the aphelion, let $\theta = 0$ and calculate

$$r = \frac{39.439(1 - 0.25024^2)}{1 - 0.25024(-1)} = 49.308 \text{ AU}.$$

We now turn our attention to **Figure 6**, and let $2a$ be the length of the major axis of the ellipse. The closest point of the orbit to the Sun is called *perihelion* and the farthest point is called *aphelion*. Calculating their polar coordinates from the equation of the ellipse, we let $\theta = \pi$ and get

$$r = \frac{ep}{1+e} \text{ and let } \theta = 0\text{, and get } r = \frac{ep}{1-e}.$$

Then from Figure 6,

$$2a = \frac{ep}{1+e} + \frac{ep}{1-e} = \frac{2ep}{1-e^2}.$$

This means that $ep = a(1 - e^2)$ and the polar equation of the elliptical orbit can be written in the form

$$r = \frac{a(1-e^2)}{1 - e\cos\theta}.$$

Do the following You Try Its. Carefully label all answers and indicate units. Round to the nearest million miles, and to the nearest ten-thousandth AU. Use 1 AU = 92,900,000 miles. ❑

You Try It #1

a. Find the polar equation of the orbit of the planet Mercury.
b. Find its distance from the Sun at perihelion and at aphelion in AU.
c. Find these distances in miles.

You Try It #2

a. Find the polar equation of the orbit of Earth.
b. Find its distance from the Sun at perihelion and at aphelion an AU and in miles.
c. How close does the Earth get to the Sun?

You Try It #3

a. Draw by hand, graphing package, or graphing calculator, a graph containing the orbits of Earth and Mercury and label points and give distances in miles.
b. How far in miles is it between the perihelion of Earth and Mercury?
c. What is the length of the major axis of the orbit of Earth?
d. Which of the two has the "largest orbit," the most elliptical orbit?

You Try It #4

Referring to your work in **You Try It #2** and to **Examples 1** and **2**, compare the orbits of Earth and Pluto.
a. Draw by hand a rough graph with their orbits and label points and give distances in miles.
b. Give the length of the semimajor axis for Pluto in miles.
c. How many miles does Pluto get from the Sun?
d. How much "larger" is the orbit of Pluto than that of Earth? Answer this question several ways.

You Try It #5

As Halley's comet orbits the Sun, it is about 900,000 km from the Sun at perihelion and 5,340,000 km at aphelion. Calculate the length of the major axis, the minor axis, and the distance between the foci.

You Try It #6

Asteroids are small irregularly shaped objects in space. Over 1570 have been identified. Ceres, the largest is about 500 miles in diameter.

Most of the asteroids are between the orbits of Jupiter and Mars. Use the solar system chart to explain if this is true for the Asteroid Icarus. Draw by hand or by computer graph the orbits of the three, and label points and give distances in miles.

You Try It #7

Kepler's third law states that the period of a planet (the time to make one complete revolution) is proportional to the length of the semimajor axis raised to the $^3/_2$ power.
a. Use the Solar System Chart to build a table of the periods of all the planets listed.
b. Why is the period of the Earth equal to one? What is this time interval called by Earthlings? How many years does it take Pluto to complete its orbit?

You Try It #8

Estimate how far the Earth travels in its planar orbit in one year. What is the average speed of Earth in miles per hour?

You Try It #9

If a body were found orbiting the Sun with a period of 292 days, what would be the semimajor axis of its orbit? Which planet would pass nearest to it?

You Try It #10

To read about astronomy software for personal computers which let you see the comets approach and recede, see eclipses, and puts you in control of the universe, see "Exploring the Sky on Computer," Sky and Telescope, June, 1990, by John Mosely.

Use a computer graphing package or a graphing calculator to construct a graph containing the Sun and the parts of the orbits of Kahotek, Halley, and Earth, which include perihelion of each. Report your device, program, range, equations (polar, rectangular, or parametric), and give the graph. Which of these bodies comes closest to the Sun? How close is perihelion of Halley to that of Earth, if their orbits were coplanar?

You Try It #11

Write the equation for the elliptical orbit of Mars in the rectangular form

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \text{ with major vertices at } (\pm a, 0),\ c = ae\text{, and } b^2 = a^2 - c^2.$$

You Try It #12

For the planet Mars, what is the equation of its other directrix (the one farthest from the Sun)? (See **Figure 5**.) Where is the other focus? What is at the other focus?

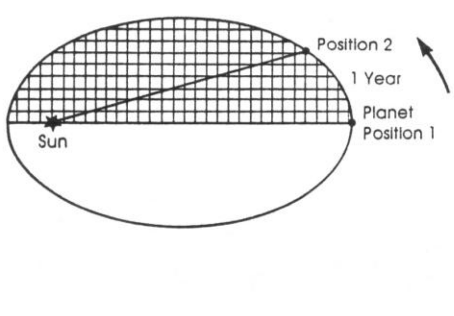
You Try It #13

Use the rectangular equation of the orbit of Mars from the above **You Try It #11** to construct the graph by using computerized graphing. Report your device, equations, programs, range, and graph.

You Try It #14

Figure 7:

Elliptical orbit of a hypothetical celestial object.



Kepler's second law of planetary motion states that each planet moves in such a way that the imaginary line connecting it to its sun sweeps out equal areas in equal intervals of time. **Figure 7** depicts the elliptical orbit of a hypothetical celestial object. A grid is provided and can be used to estimate the area swept out by a line connecting the sun and the planet as the planet moves from Position 1 to Position 2. Determine the number of boxes between these lines. Now locate a third position by shifting counterclockwise the position until the total number of boxes between Position 2 and Position 3 is approximately equal to the number of boxes (area) you found between Position 1 and Position 2. Assuming equal time intervals of one "year" for each interval, what must be true of the orbital velocity of the planet?

Some Answers to the You Try Its

1

a Mercury: $$r = \frac{0.3871\,(1 - 0.20563^2)}{1 - 0.20563 \cos\theta}.$$

b, c The distance from the Sun at perihelion = 0.3075 AU = 28,600,000 miles. The distance from the Sun at aphelion = 0.4667 AU = 43,400,000 miles.

2

a Earth: $$r = \frac{1 - 0.01673^2}{1 - 0.01673 \cos\theta}.$$

b The distance from the Sun at perihelion = 90,500,000 miles. The distance at aphelion = 93,500,000.

3

b The distance between perihelion of Earth and Mercury is 61,900,000 miles.

d Mercury has the most elliptical orbit.

4

c The greatest distance of Pluto from the Sun is 4.5807 billion miles.

d The orbit of Pluto is 39 times "larger" than that of Earth. The major axis of Pluto is 7,142,000,000 miles longer than that of Earth.

5 The length of the major axis for Halley is 6,240,000 km. The length of its minor axis is 3.9755 AU. The distance between the foci is 4,440,000 km.

6 The orbit of the Asteroid *Icarus* is between that of Jupiter and Mars because the length of its semimajor axis is between that of Jupiter and Mars, they all have the Sun at a common focus, and their orbits are nearly circular.

10 The following program with range, and parametric equations for orbits of Earth, Comet Halley, and Comet Kahotek is for a TI–81 programmable graphing calculator:

Prgm2:EARHALKA: " ($1-.01673^2$)
cosT/(1–.01673cosT) " → X_{1T}: "($1-.01673^2$)
sinT/(1–.01673cosT) " → Y_{1T}: "36.178($1-.97214^2$)
cosT/(1–.97214cosT) " → X_{2T}: " 36.178 ($1-.97214^2$)
sinT/1–.97214 cosT " → Y_{2T}: "3466.7 ($1-.99993^2$
cosT/(1–.99993 cosT) " → X_{3T}: "3466.7 ($1-.99993^2$)
sinT/(1–.99993cosT) " → Y_{3T}: → 1.5 → Xmin:1.5
→Xmax:1 → Xscl:–1 → Ymin: 1 → Ymax: 1 → Yscl:0
→ Tmin: 6.2834 → Tmax: .10473 → Tstep: DispGraph

When the program is executed, the graph looks similar to the one below. The perihelion of Kahotek is closest to the Sun. Perihelion of Comet Halley is close to that of the Earth (about 2,000,000 miles). The orbits of these three objects are in different intersecting planes, although for graphical purposes, we have plotted them in the same plane.

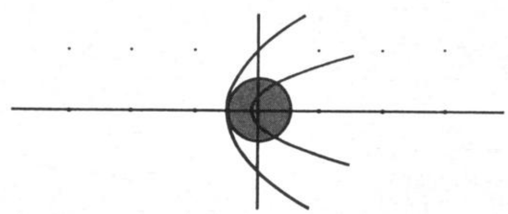


Figure 8:

Orbits of Earth, Comet Halley, and Comet Kahotek.

11
$$e = \frac{c}{a},$$
$a = 1.5237$ AU,
$c = 0.09337\ (1.5237)$,
$b^2 = a^2 - c^2 = 2.3014$,
$$\frac{x^2}{1.5237^2} + \frac{y^2}{1.5170^2} = 1.$$

14 The orbital velocity of the planet is increasing. As the planet gets closer to its sun, the gravitational force increases, thus the acceleration increases with the result being maximum orbital velocity at perihelion.