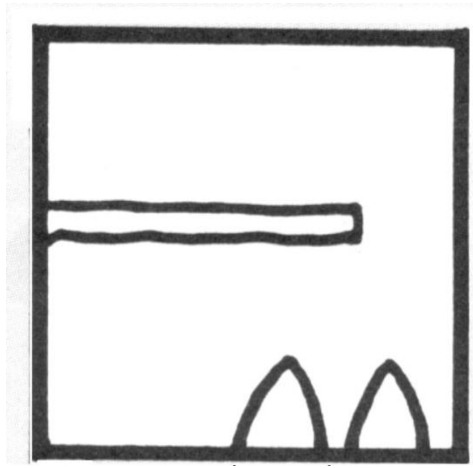
Turvy with Integration -- A Puzzle by David Pleacher



Back in 1953, Roger Price invented a minor art form called the Droodle, which he described as "a borkley-looking sort of drawing that doesn't make any sense until you know the correct title." In 1985, *Games* Magazine took the Droodle one step further and created the Turvy. Turvies have one explanation right-side-up and an entirely different one turned topsy-turvy. The Turvy above was created by P. Edwin Letcher of Los Angeles, California and published in *Games* Magazine in May 1986.

Here is the title right-side-up:

"— — — — — — — — — — — — — — — — — — — — — ."

15 9 19   20 19 20 18 14   17 15   17   5 17 11 17 5 18   14 17 8 18

Here is the title upside-down:

"— — — — — — — — — — — : — — — — — — — — — — —

19 8 4 6 20 16 12   7 16 3 18   17   20 18 11 2 18 12 15   15 18 10

— — — — — — — — — — — — ."

2 19 11   15 19 18   20 19 16 10 15 14

To determine the titles to this turvy, solve the 20 integral problems on the following pages. Then replace each numbered blank with the letter corresponding to the answer for that problem.

Integral Problems: Answers:

1.	$\int x \ln x \, dx =$	A.	$\arcsin\frac{x}{2} + C$
2.	What is the area under the curve described by the parametric equations $x = \sin t$ and $y = \cos^2 t$ for $0 \le t \le \frac{\pi}{2}$?	B.	$\frac{1}{4}$
3.	$\int_1^e \left(\frac{x^2+4}{x}\right) dx =$	C.	-2 and 2
4.	$\int 6x^3 e^{3x} dx =$	D.	I, II, and III
5.	$\int_0^1 e^{2x} \, dx =$	E.	$\frac{-1}{4}\operatorname{arcsec}\left(\frac{\lvert\cot x\rvert}{4}\right) + C$
6.	If $F(x) = \int_2^{x^2} t^2 \, dt$, Then $F(2) =$	F.	$\frac{2}{3}$
7.	Which of the following are antiderivatives of $f(x) = \cos^3 x \sin x$? I. $F(x) = \frac{-\cos^4 x}{4}$ II. $F(x) = \frac{\sin^2 x}{2} - \frac{\sin^4 x}{4}$ III. $F(x) = \frac{1-\cos^4 x}{4}$	G.	$\frac{e^2-1}{2}$
8.	$\int_0^{\frac{\pi}{4}} \sin 2x \, dx =$	H.	I and III

Integral Problems:

9. $\int_0^{\pi/3} \left(\frac{\tan x \, e^{\sec x}}{\cos x} \right) dx =$

10. $\int_0^1 \left(\sqrt{x} \right) \left(x^2 + 3x - 8 \right) dx =$

11. $\int_1^5 \left(\frac{3x}{x^3} \right) dx =$

12. What are all the values of k such that $\int_{-2}^{k} x^3 \, dx = 0$?

13. $\int_0^{\pi/4} \sin x \cos x \, dx =$

14. $\int x \sec^2 x \, dx =$

15. $\int_1^9 \frac{e^{\sqrt{x}}}{\sqrt{x}} dx =$

16. $\int e^x \sin x \, dx =$

17. $\int \frac{4 \, dx}{\sqrt{64 - 16x^2}} =$

18. $\int \frac{\tan x \, dx}{\sin^2 x \sqrt{\cot^2 x - 16}} =$

19. $\int \frac{e^{\tan x}}{1 - \sin^2 x} dx =$

Answers:

I. $\frac{1}{2} e^x (\sin x - \cos x) + C$

J. I only

K. $\frac{x^2}{4} (2 \ln x - 1) + C$

L. $\frac{1}{2}$

M. $\frac{56}{3}$

N. $\frac{-404}{105}$

O. $e^{\tan x} + C$

P. -42

Q. -18

R. $\frac{12}{5}$

S. $x \tan x + \ln|\cos x| + C$

Integral Problems:

20. $\begin{cases} \text{If } \int_0^3 f(x)dx = 10 \text{ and } \int_3^0 g(x)dx = 12 \\ \text{Then evaluate } \int_0^3 \left(g(x) - 3f(x)\right)dx \end{cases}$

Answers:

T. $2e\left(e^2 - 1\right)$

U. $\frac{2}{9}e^{3x}\left(9x^3 - 9x^2 - 6x - 2\right) + C$

V. $\frac{e^2 + 7}{2}$

W. $\mathrm{e}^2 - e$

X. $\frac{1}{2}e^x\left(\sin x + 2\cos x\right) + C$

Y. $\frac{2}{9}e^{3x}\left(9x^3 - 9x^2 + 6x - 2\right) + C$

Z. None of the Above

Thanks to Rodger Jaffe and his class for finding an error in the puzzle!