Mini Examination #2    A.P. Calculus    Name _____

I. Multiple Choice

_____ 1. Determine the differential $dy$ of $y = 3x^5 - 5x^2 + 6$

(A) $\left(15x^4 - 10x\right)dx$        (B) $\left(15x^4 - 10x + 1\right)dx$        (C) $15x - 10$

(D) $15x^4 - 10x$     (E) None of the above

_____ 2. What is the instantaneous rate of change of $f(x) = x^3 - 3x^2 + x - 1$ at $x = 2$?

(A) 13     (B) 1     (C) 0     (D) $-5$     (E) 2

_____ 3. Determine $\dfrac{dy}{dx}$ for the curve defined by $x^3 + y^3 = 3xy$

(A) $\dfrac{x^2}{y^2 - x}$     (B) $\dfrac{x^2}{x - y^2}$     (C) $\dfrac{y - x^2}{y^2 - x}$     (D) $\dfrac{1 - x}{y - 1}$     (E) $\dfrac{x^2 - y}{y^2 - x}$

_____ 4. $\lim\limits_{x \to \infty}\left(\dfrac{7x^2 + 5x - 3}{2 + 3x - 11x^2}\right) =$

(A) $-\dfrac{7}{2}$     (B) $-\dfrac{5}{11}$     (C) $-\dfrac{7}{11}$     (D) $\dfrac{7}{11}$     (E) It is nonexistent

_____ 5. Water is draining at the rate of $48\pi$ cubic feet per minute from an inverted conical tank whose diameter at its base is 40 feet and whose height is 60 feet. How fast is the height of the water in the tank decreasing at the instant that the height is 48 feet?

(A) 3 ft/min     (B) $\dfrac{3}{2}$ ft/min     (C) $\dfrac{2}{3}$ ft/min     (D) $\dfrac{3}{16}$ ft/min     (E) $\dfrac{1}{16}$ ft/min

_____ 6. If $f(x) = \tan\left(e^{\sin x}\right)$, then $f'(x) =$

(A) $-e^{\sin x} \cos x \sec^2\left(e^{\sin x}\right)$

(B) $e^{\sin x} \cos x \sec^2\left(e^{\sin x}\right)$

(C) $-e^{\sin x} \sec\left(e^{\sin x}\right) \tan\left(e^{\sin x}\right)$

(D) $e^{\sin x} \sec^2\left(e^{\sin x}\right)$

(E) $e^{\sin x} \sec\left(e^{\sin x}\right) \tan\left(e^{\sin x}\right)$

_____ 7. The normal line to the curve $y = \sqrt{8 - x^2}$ at the point (2, 2) has slope

(A) $-2$     (B) $-\dfrac{1}{2}$     (C) $\dfrac{1}{2}$     (D) 1     (E) 2

_____ 8. Evaluate the limit:    $\displaystyle\lim_{h \to 0}\left(\dfrac{4\left(\dfrac{1}{2} + h\right)^8 - 4\left(\dfrac{1}{2}\right)^8}{h}\right) =$

(A) 0     (B) $\dfrac{1}{256}$     (C) $\dfrac{1}{64}$     (D) $\dfrac{1}{2}$     (E) $\dfrac{1}{4}$

_____ 9. If $y = e^{3x} \sin x^2$ then $f'(x) =$

(A) $e^{3x} \cos x^2$          (B) $e^{3x} \cos x^2 + e^{3x} \sin x^2$          (C) $6xe^{3x} \cos x^2$

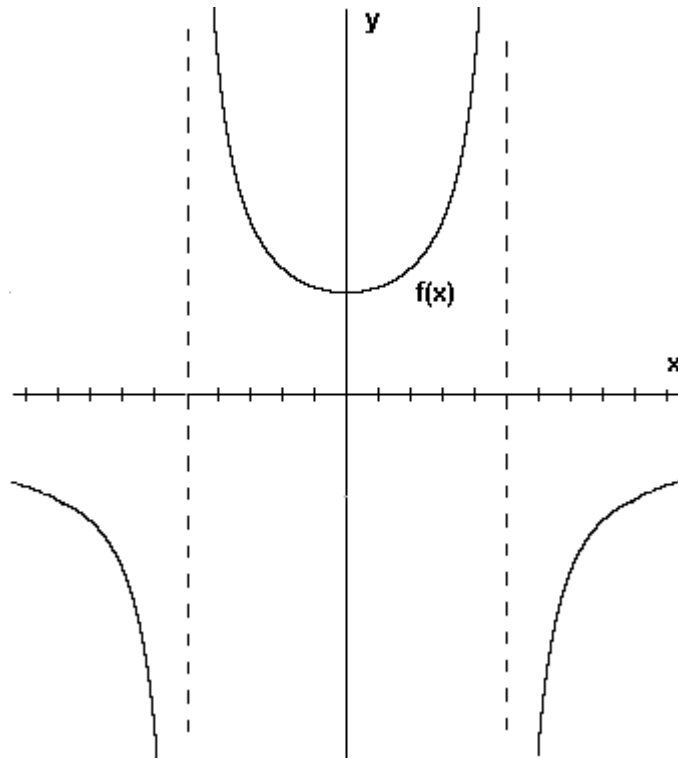(D) $2e^{3x} \cos x^2$          (E) $2xe^{3x} \cos x^2 + 3e^{3x} \sin x^2$

II. Free Response
   SHOW ALL WORK ON YOUR OWN PAPER. Do NOT write on this test paper.

10. Determine the $38^{th}$ derivative of $y = \sin 2x$.

11. Determine the second derivative of $y = \tan x$.

12. Draw the graph of the ***derivative*** of the function below:



13. Given the parametric equations:  $\begin{aligned} y &= 5t^2 + 1 \\ x &= t - 1 \end{aligned}$

   Determine the derivative $\dfrac{dy}{dx}$

14. Evaluate the limit: $\displaystyle\lim_{x\to 0}\frac{2\sin 10x}{x} =$

15. Given $f(1)=1$, $g(1)=-2$, $f'(1)=3$, and $g'(1)=-1$,

   Evaluate $\dfrac{d}{dx}\left(\dfrac{f(x)}{g(x)}\right)\Big|_{x=1}$

16. Determine the value of $k$ if the graphs of $y = 2x-1$ and $y = x^2 - 4x + k$ are tangent to each other at a point in the first quadrant.

17. State and prove the derivative formula for $y = \tan x$.

18. Determine the derivative of $h(x) = 3x^2 \sin^3(2x)$.
    SHOW ALL STEPS!