Mini Exam #3 A. P. Calculus Name _____

I. Multiple Choice

_____ 1. Which of the following expressions would evaluate the definite integral $\int_{-4}^{6} f(x)\,dx$, given that $f(x) = \begin{cases} |x-2|\,dx & \text{if } x \geq 0 \\ x+2 & \text{if } x < 0 \end{cases}$

(A) $\int_{-4}^{6} (x-2)\,dx$

(B) $\int_{-4}^{0} (x+2)\,dx + \int_{0}^{6} (x-2)\,dx$

(C) $\int_{-4}^{0} (x+2)\,dx + \int_{0}^{2} (x-2)\,dx + \int_{2}^{6} (-x+2)\,dx$

(D) $\int_{-4}^{-2} (x+2)\,dx + \int_{-2}^{0} (x-2)\,dx + \int_{0}^{6} (2-x)\,dx$

(E) $\int_{-4}^{0} (x+2)\,dx + \int_{0}^{2} (2-x)\,dx + \int_{2}^{6} (x-2)\,dx$

_____ 2. The acceleration of an object moving along the x-axis is given by $a(t) = 18t - 2$, where the velocity is 12 m / sec when t = 2 sec and the position is 2 meters when t = 1 second. The position $x(t)$ =

(A) $9t^3 - t^2 - 20t + 14$

(B) $9t^3 - t^2 + 20t - 26$

(C) $3t^3 - t^2 - 20t + 20$

(D) $3t^3 - t^2 + 20t - 20$

(E) $t^3 - t^2 - 20t + 22$

_____ 3. If $f(x) = \ln\left(\ln(1-x)\right)$, Then $f'(x) =$

(A) $-\dfrac{1}{\ln(1-x)}$

(B) $\dfrac{1}{(1-x)\ln(1-x)}$

(C) $\dfrac{1}{(1-x)^2}$

(D) $-\dfrac{1}{(1-x)\ln(1-x)}$

(E) $-\dfrac{1}{\ln(1-x)^2}$

_____ 4. If $\int_{-3}^{2} f(x)\,dx = 6$ and $\int_{2}^{1} f(x)\,dx = 11$, then $\int_{-3}^{1} f(x)\,dx =$

(A) 5
(B) 17
(C) 3
(D) -5
(E) 9

_____ 5. The average value of $\sqrt{x}$ over the interval $0 \le x \le 2$ is

(A) $\dfrac{\sqrt{2}}{3}$

(B) $\dfrac{\sqrt{2}}{2}$

(C) $\dfrac{2\sqrt{2}}{3}$

(D) 1

(E) $\dfrac{4\sqrt{2}}{3}$

_____ 6. If $\int_0^k \left(2kx - x^2\right) dx = 18$ Then $k =$

(A) -9
(B) -3
(C) 3
(D) 9
(E) 18

_____ 7. $\int \left(\frac{x^2 + 1}{x} \right) dx =$

(A) $\frac{1}{2}\ln\left(^2+1\right) + C$

(B) $\frac{2}{\mathrm{x}} + C$

(C) $\frac{\mathrm{x}^3}{3} + \ln|x| + C$

(D) $\frac{2\mathrm{x}}{3} + \frac{2}{x} + C$

(E) $\frac{x^2}{2} + \ln|x| + C$

_____ 8. The function $f(x) = xe^x$ has inflection points at:

(A) 0
(B) –1
(C) –2
(D) 1
(E) There are no inflection points of *f*.

_____ 9. If $\frac{dy}{dx} = \frac{4}{2x-3}$ and $y(2) = 1$, then $y =$

(A) $2\ln|2x-3|+1$

(B) $\frac{4x}{x^2 - 3x + 1} + 1$

(C) $2\ln|2x-3|$

(D) $e^{2x-3}+1$

(E) $\frac{1}{2}\ln|2x-3|-1$

_____ 10. Evaluate $\lim_{h\to 0}\left(\frac{5\left(\frac{1}{2}+h\right)^4-5\left(\frac{1}{2}\right)^4}{h}\right)$

(A) $\frac{5}{2}$

(B) $\frac{5}{16}$

(C) 40

(D) 160

(E) The limit does not exist

II. Free Response

Show all work on your own paper.

11. The Mean Value Theorem guarantees the existence of a special point on the graph of $f(x)=x^2-x-12$ on the interval [– 3, 4]. What are the coordinates of this point?

12. A circle is increasing in area at the rate of 16π in^2/sec. How fast is the radius increasing when the radius is 2 in?

13. An open rectangular box is made from a square piece of metal (each side of length 24 inches) by cutting out square corners and folding up the sides. What size corners should be cut to maximize the volume of the box?

14. Determine the area between the curve $y=x^2-4$ and the x-axis from $x=0$ to $x=4$.

15. If $x^2-2xy+3y^2=8$, Then $\frac{dy}{dx}=$

16. For what value of k will $y=x+\frac{k}{x}$ have a relative maximum at $x=-2$?

17. Evaluate the following derivative: $\dfrac{d}{dx}\left(\int_{3}^{x} \cos t\, dt\right)$

18. Determine the value of x on the closed interval $[-2,4]$ for which the function $y = x^3 - 3x^2 + 12$ has its absolute maximum.

19. Determine the derivative of $y = Cos^{-1}(5x)$.

20. Evaluate the indefinite integral: $\int \sec^3\left(4x^2\right)\tan\left(4x^2\right) x\, dx$