Mini Exam #3 A. P. Calculus Name ____________________________

I. Multiple Choice

_____ 1. The radius of a circle is decreasing at a rate of 2 cm / sec. Find the rate of change of the area of the circle with respect to time (in cm^2 / sec).

(A) $-4\pi r$
(B) $-2\pi r$
(C) 2
(D) $2\pi r$
(E) $4\pi r$

_____ 2. An object has a velocity $v(t) = 3t^4 - 3t^2$, where $t \geq 0$. Find an expression for the total distance traveled by the object between time t = 0 and time t = 4.

(A) $\int_0^1 \left(3t^4 - 3t^2\right) dt + \int_1^4 \left(3t^2 - 3t^4\right) dt$
(B) $\int_0^2 \left(3t^2 - 3t^4\right) dt + \int_2^4 \left(3t^4 - 3t^2\right) dt$
(C) $\int_0^1 \left(3t^2 - 3t^4\right) dt + \int_1^4 \left(3t^4 - 3t^2\right) dt$
(D) $\int_0^4 \left(3t^4 - 3t^2\right) dt$
(E) $\int_0^4 \left(\frac{3t^5}{5} - t^3\right) dt$

_____ 3. The second derivative with respect to x of $f(x) = e^x + e^{-x}$ is:

(A) $e^x - e^{-x}$
(B) $2f(-x)$
(C) $f(x)$
(D) $-f(x)$
(E) 0

______ 4. If $\int_{30}^{100} f(x)\,dx = A$ and $\int_{50}^{100} f(x)\,dx = B$, Then $\int_{30}^{50} f(x)\,dx =$

(A) $A + B$
(B) $B - A$
(C) 0
(D) $A - B$
(E) 20

______ 5. The average value of the function $f(x) = (x-1)^2$ on the interval from $x = 1$ to $x = 5$ is

(A) $-\frac{16}{3}$
(B) $\frac{16}{3}$
(C) $\frac{64}{3}$
(D) $\frac{66}{3}$
(E) $\frac{256}{3}$

______ 6. $\int_{0}^{1} (t^2 + 1)^2\,dt =$

(A) $\frac{28}{15}$
(B) $\frac{4}{3}$
(C) $\frac{7}{3}$
(D) $\frac{23}{15}$
(E) $\frac{6}{5}$

_____ 7. $\int x\sqrt{5x^2-4}\,dx=$

(A) $\frac{1}{10}\left(5x^2-4\right)^{\frac{3}{2}}+C$

(B) $\frac{1}{15}\left(5x^2-4\right)^{\frac{3}{2}}+C$

(C) $-\frac{1}{5}\left(5x^2-4\right)^{-\frac{1}{2}}+C$

(D) $\frac{20}{3}\left(5x^2-4\right)^{\frac{3}{2}}+C$

(E) $\frac{3}{20}\left(5x^2-4\right)^{\frac{3}{2}}+C$

_____ 8. Which of the following expressions would evaluate the definite integral $\int_{-4}^{6} f(x)\,dx$, given that $f(x)=\begin{cases} |x-2|\,dx & \text{if } x\geq 0 \\ x+2 & \text{if } x<0 \end{cases}$

(A) $\int_{-4}^{6}(x-2)\,dx$

(B) $\int_{-4}^{0}(x+2)\,dx+\int_{0}^{6}(x-2)\,dx$

(C) $\int_{-4}^{0}(x+2)\,dx+\int_{0}^{2}(x-2)\,dx+\int_{2}^{6}(-x+2)\,dx$

(D) $\int_{-4}^{-2}(x+2)\,dx+\int_{-2}^{0}(x-2)\,dx+\int_{0}^{6}(2-x)\,dx$

(E) $\int_{-4}^{0}(x+2)\,dx+\int_{0}^{2}(2-x)\,dx+\int_{2}^{6}(x-2)\,dx$

______ 9. Suppose $\frac{dy}{dt} = t(y+1)$ and $y = 0$ when $t = 0$.

Find an explicit expression for $y(t)$ as a function of t.

(A) $y(t) = e^{\frac{t^2}{2}} - 1$

(B) $\ln|y(t)+1| = \frac{t^2}{2}$

(C) $y(t) = 1 - e^t$

(D) $y(t) = 2e^{\frac{t^2}{2}}$

(E) $y(t) = t^2 + e^t - 1$

______ 10. What is $\lim_{h \to 0} \frac{\cos\left(\frac{\pi}{2}+h\right) - \cos\left(\frac{\pi}{2}\right)}{h}$?

(A) $-\infty$

(B) -1

(C) 0

(D) 1

(E) ∞

II. Free Response

Show all work on your own paper.

11. An open rectangular box is made from a square piece of metal (each side of length 24 inches) by cutting out square corners and folding up the sides. What size corners should be cut to maximize the volume of the box?

12. The Mean Value Theorem guarantees the existence of a special point on the graph of $f(x) = x^2$ on the interval [1, 3]. What are the coordinates of this point?

13. Given $x^2y + y^2 = 5$, determine $\frac{dy}{dx} =$

14. Determine the area between the curve $y = x^2 - 1$ and the x-axis from $x = 0$ to $x = 4$.

15. For what value of x does the function $f(x) = x^3 - 9x^2 - 120x + 6$ have a relative minimum?

16. Determine the point of inflection of $y = 5x^4 - x^5$.

17. Determine the derivative of $y = Tan^{-1}(6x)$.

18. Evaluate the indefinite integral: $\int \sec^5(3x^2)\tan(3x^2)\,x\,dx$

19. Evaluate the following derivative: $\frac{d}{dx}\left(\int_2^x \sin t\,dt\right)$

20. Determine the value of x on the closed interval $[-2, 4]$ for which the function $y = x^3 - 3x^2 + 12$ has its absolute maximum.