A. P. Calculus First Semester Examination Mr. Pleacher Name ____________________

A calculator is allowed on this section of the Exam.

I. Multiple Choice

_____ 1. The slope of the line tangent to the graph of $y = \ln\left(x^2\right)$ at $x = e^2$ is

(A) $\frac{1}{e^2}$ (B) $\frac{2}{e^2}$ (C) $\frac{4}{e^2}$ (D) $\frac{1}{e^4}$ (E) $\frac{4}{e^4}$

_____ 2. With respect to the y-axis, point $(-4, 1)$ is symmetric to

A $(4, 1)$ B $(-4, -1)$ C $(4, -1)$ D $(1, -4)$ E $(1, 4)$

_____ 3. Determine the differential dy given that $y = x\cot(x)$

(A) $-\csc^2 x$ (B) $-x\csc^2 x\,dx$ (C) $-x\csc^2 x + \cot x$

(D) $\left(-x\csc^2 x + \cot x\right)dx$ (E) None of these

_____ 4. Determine the equation of the tangent line to the graph of $y = f(x)$ at the point where $x = -3$ if $f(-3) = 2$ and $f\,'(-3) = 5$.

(A) $y = -3$ (B) $y = 5x + 2$ (C) $y = 5x + 17$ (D) $y = 2x + 5$

(E) It can not be determined from this information.

_____ 5. The derivative of $y = 6x^2$ is

(A) x (B) 12 (C) $12x$ (D) 0 (E) None of these

_____ 6. The second derivative of $y = 8x$ is

(A) 0 (B) $8x$ (C) x (D) 8 (E) None of these

_____ 7. A ball is dropped from a height of 1 meter. It always bounces to one-half its previous height. The ball will bounce infinitely but it will travel a finite distance. What is the distance?

(A) 4 m (B) 3 m (C) $2\ \dfrac{31}{32}$ m (D) 2 m (E) It can not be determined

_____ 8. $\dfrac{d}{dx}\left(Sin^{-1}(2x)\right) =$

(A) $\dfrac{-1}{2\sqrt{1-4x^2}}$ (B) $\dfrac{-2}{\sqrt{4x^2-1}}$ (C) $\dfrac{1}{2\sqrt{1-4x^2}}$

(D) $\dfrac{2}{\sqrt{1-4x^2}}$ (E) $\dfrac{2}{\sqrt{4x^2-1}}$

_____ 9. If $y = e^{nx}$, where n is a constant, then $\dfrac{d^n y}{dx^n} =$

(A) $n^n e^{nx}$ (B) $n!\,e^{nx}$ (C) $n e^{nx}$ (D) $n^n e^{nx-1}$ (E) $n!\,e^{x}$

_____ 10. Water flows at 8 cubic feet per minute into a cylinder with radius 4 feet. How fast is the water level rising?

(A) 2 ft/min (B) $\dfrac{1}{\pi}$ ft/min (C) $\dfrac{1}{2\pi}$ ft/min (D) 2π ft/min

(E) None of the above

_____ 11. The value of $\dfrac{d^2y}{dx^2}$ in the equation $y^2 + y = x$ at the point (2, 1) is:

(A) $\dfrac{-2}{27}$ (B) $\dfrac{1}{3}$ (C) $\dfrac{1}{5}$ (D) $\dfrac{-2}{125}$ (E) $\dfrac{1}{2}$

______ 12. If the graph of $y = ax^3 + 4x^2 + cx + d$ has a point of inflection at $(1,0)$, then the value of a is:

(A) 2 (B) $-\frac{4}{3}$ (C) $\frac{1}{2}$ (D) $\frac{8}{3}$ (E) None of these

______ 13. The equation of the normal line to the curve $y = x^4 + 3x^3 + 2$ at the point where $x = 0$ is

(A) $y = x$ (B) $y = 13x$ (C) $y = 0$ (D) $y = x + 2$ (E) $x = 0$

______ 14. If $y = \sin u$, $u = 3w$, and $w = e^{2x}$, then $\frac{dy}{dx} =$

(A) $6e^{2x}\cos\left(3e^{2x}\right)$ (B) $3\cos\left(e^{2x}\right)$ (C) $e^{2\cos\left(3e^{2x}\right)}$ (D) $-6\sin\left(6e^{2x}\right)$

(E) $6x\cos\left(e^{2x}\right)$

______ 15. $\frac{d}{dx}\left(\text{Arccos } 3x\right) =$

(A) $\frac{3}{\sqrt{1-x^2}}$ (B) $\frac{-1}{3\sin 3x}$ (C) $\frac{3}{\sqrt{1-3x^2}}$ (D) $\frac{-3}{\sqrt{1-9x^2}}$

(E) $\frac{3}{\sqrt{9x^2-1}}$

______ 16. If $f(x) = 2e^x + e^{2x}$, then $f'''(0) =$

(A) 10 (B) 8 (C) 6 (D) 4 (E) 3

______ 17. Determine the absolute maximum value and the absolute minimum value of the function $f(x) = 2x^3 + 3x^2 - 12x$ over the interval $[-3, 3]$.

(A) Absolute Maximum value is 20; Absolute Minimum value is -7

(B) Absolute Maximum value is 45; Absolute Minimum value is -7

(C) Absolute Maximum value is 3; Absolute Minimum value is 1

(D) Absolute Maximum value is -2; Absolute Minimum value is 1

(E) Absolute Maximum value is 45; Absolute Minimum value is 9

______ 18. Given L feet of fencing, what is the maximum number of square feet that can be enclosed if the fencing is used to make three sides of a rectangular pen, using an existing wall as the fourth side?

(A) $\dfrac{L^2}{4}$ (B) $\dfrac{L^2}{8}$ (C) $\dfrac{L^2}{9}$ (D) $\dfrac{L^2}{16}$ (E) $\dfrac{2L^2}{9}$

______ 19. If $y = \ln(\sin x)$, then $\dfrac{dy}{dx} =$

(A) $\dfrac{1}{\sin x}$ (B) $\ln(\sin x)$ (C) $\cot x$ (D) $\dfrac{1}{x}\sin x + \ln(\cos x)$

(E) $(\cos x)\ln(\sin x)$

______ 20. A function f is defined by $f(x) = \begin{cases} \dfrac{x^2 - 4}{x - 2} & \text{if } x \neq 2 \\ k & \text{if } x = 2 \end{cases}$

If f is continuous at $x = 2$, what is the value of k?

(A) 4 (B) -2 (C) 2 (D) 0 (E) $\dfrac{1}{2}$

II. Free Response

21. Prove the following derivative formula:

Given: $y = e^x$

Prove: $\frac{dy}{dx} =$

22. Using the delta-epsilon definition of a limit, Prove that the $\lim_{x \to 3} 2x + 4 = 10$

23. Write out the complete definition for continuity.

________________ 24. Determine $\frac{dy}{dx}$, given that $x^2 + y^2 = 16$

________________ 25. Determine $\frac{dy}{dx}$, given that $xy + x^2 = 1607$

________________ 26. Determine $\frac{dy}{dx}$, given that $y = \left(4x^2 - 5\right)^{10}$

________________ 27. Determine $\frac{dy}{dx}$, given that $y = \frac{x^2}{\cos(x)}$

________________ 28. Determine $\frac{dy}{dx}$, given that $x = 3t + 1$ and $y = t^2 + t$

________________ 29. Determine the domain of $y = \sqrt{\dfrac{x}{x+4}}$

________________ 30. Determine the equation of the tangent to the curve $\sin(y) = \cos(x)$ at the point $\left(\dfrac{\pi}{2}, 0\right)$.

________________ 31. A particle projected vertically upward with an initial velocity of 256 ft/sec reaches an elevation $s = 256t - 16t^2$ at the end of t seconds. How high does the particle rise?

________________ 32. For $x \neq 4$, the function $h(x) = \dfrac{x^2 + x - 20}{x - 4}$. What value should be assigned to $h(4)$ to make $h(x)$ continuous at $x = 4$?

________________ 33. Determine the value of x if $f(x) = x^2$ and

$$f'(f(x)) + f(f'(x)) = 54.$$

________________ 34. If $y = \sin(2x)$, determine the 50^{th} derivative of y with respect to x.

a = _____________ 35. The curve $y = ax^2 + bx + c$ passes through the points $(2, 5)$ and

b = _____________ $(-2, -3)$. The value of y is greatest when $x = 1$. Determine the

c= ______________ values of a, b, and c.

________________ 36. Solve for y *(to three decimal places)*: $8^y = 77$

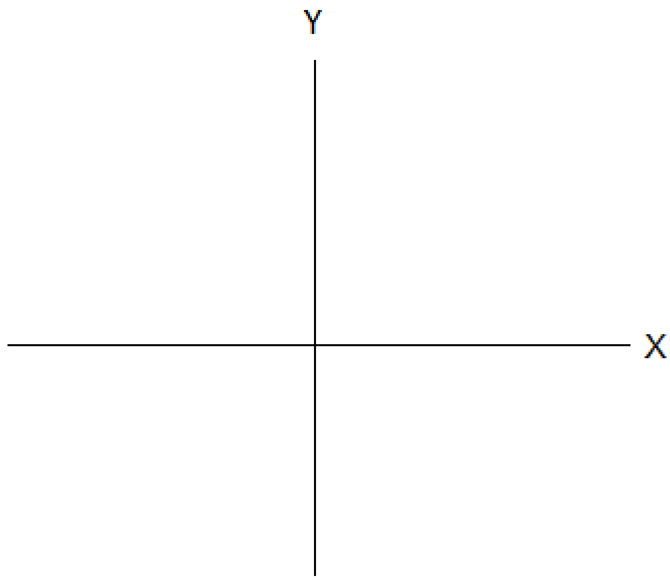
________________ 37. Grain pouring from a chute at the rate of $6\ ft^3/\min$ forms a conical pile whose altitude is always twice its radius. How fast is the altitude of the pile changing when the radius is 4 feet?

38. Sketch a curve which satisfies the following conditions:

$\frac{dy}{dx} > 0$ on $(-\infty, 0)$ and $(2, +\infty)$ $\qquad$ $\frac{dy}{dx} < 0$ on $(0, 2)$

$\frac{d^2y}{dx^2} > 0$ on $(1, +\infty)$ $\qquad$ $\frac{d^2y}{dx^2} < 0$ on $(-\infty, 1)$

$f(0) = 4$ $\qquad$ $f(2) = 0$ $\qquad$ $f(1) = 1$



_______________ 39. Given $f(1) = 2$, $f'(1) = 4$, and $g(x) = \left(f(x)\right)^{-3}$

Determine $\frac{d}{dx}\left(g(x)\right)\Big|_{x=1}$

_______________ 40. Use differentials to determine the value of $\sqrt[3]{26}$.