A. P. Calculus First Semester Examination Mr. Pleacher Name ____________________

A calculator is allowed on this section of the Exam.

I. Multiple Choice

_____ 1. If $y^2 x = 4$, then $\dfrac{dy}{dx} =$

(A) $\dfrac{x^2}{y}$ (B) $\dfrac{2}{\sqrt{x}}$ (C) $\dfrac{-y}{2x}$ (D) $\dfrac{-2y}{x}$ (E) $4x^{\frac{1}{2}}y$

_____ 2. If the graph of $y = ax^3 + 4x^2 + cx + d$ has a point of inflection at $(1,0)$, then the value of a is:

(A) 2 (B) $-\dfrac{4}{3}$ (C) $\dfrac{1}{2}$ (D) $\dfrac{8}{3}$ (E) None of these

_____ 3. The equation of the normal line to the curve $y = x^4 + 3x^3 + 2$ at the point where $x = 0$ is

(A) $y = x$ (B) $y = 13x$ (C) $y = 0$ (D) $y = x + 2$ (E) $x = 0$

_____ 4. Given $g(1) = -2,\quad g'(1) = -1,$ determine the value of $\frac{d}{dx}\left(g^3(x)\right)$ when $x = 1.$

(A) -12 (B) 0 (C) 6 (D) 12 (E) None of these

_____ 5. The derivative of $y = 12x$ is

(A) x (B) 12 (C) $12x$ (D) 0 (E) None of these

_____ 6. The second derivative of $y = 8x$ is

(A) 8 (B) $8x$ (C) x (D) 0 (E) None of these

_____ 7. A bouncing ball loses $\frac{1}{4}$ of its previous height each time that it rebounds. If the ball is thrown up to a height of 60 feet, how many feet will it travel before coming to rest (including the 60 feet on the way up)?

(A) 480 feet (B) 240 feet (C) 160 feet (D) 120 feet (E) 80 feet

_____ 8. $\frac{d}{dx}\left(Sin^{-1}(2x)\right) =$

(A) $\frac{-1}{2\sqrt{1-4x^2}}$ (B) $\frac{-2}{\sqrt{4x^2-1}}$ (C) $\frac{1}{2\sqrt{1-4x^2}}$

(D) $\frac{2}{\sqrt{4x^2-1}}$ (E) $\frac{2}{\sqrt{1-4x^2}}$

_____ 9. If $y = e^{nx}$, where n is a constant, then $\frac{d^n y}{dx^n} =$

(A) $n^n e^{nx}$ (B) $n!\, e^{nx}$ (C) $n e^{nx}$ (D) $n^n e^{nx-1}$ (E) $n!\, e^{x}$

_____ 10. Water flows at 8 cubic feet per minute into a cylinder with radius 4 feet. How fast is the water level rising?

(A) 2 ft/min (B) $\frac{1}{2\pi}$ ft/min (C) $\frac{1}{\pi}$ ft/min (D) 2π ft/min

(E) None of the above

_____ 11. The slope of the line tangent to the graph of $y = \ln\left(x^2\right)$ at $x = e^2$ is

(A) $\frac{2}{e^2}$ (B) $\frac{1}{e^2}$ (C) $\frac{4}{e^2}$ (D) $\frac{1}{e^4}$ (E) $\frac{4}{e^4}$

_____ 12. With respect to the origin, point $(-4, 1)$ is symmetric to

A $(4, 1)$ B $(-4, -1)$ C $(4, -1)$ D $(1, -4)$ E $(1, 4)$

_____ 13. If $y = \csc(h(x))$, then $\dfrac{dy}{dx} =$

(A) $-\cot^2(h(x))$ (B) $-\csc(h(x))\cot(h(x))h'(x)$

(C) $-\cot^2(h(x))\,h'(x)$ (D) $-\sec(h(x))\cot(h(x))h'(x)$

(E) None of these

_____ 14. Determine the equation of the tangent line to the graph of $y = f(x)$ at the point where $x = -3$ if $f(-3) = 2$ and $f'(-3) = 5$.

(A) $y = -3$ (B) $y = 5x + 2$ (C) $y = 2x + 5$ (D) $y = 5x + 17$

(E) It cannot be determined from this information.

_____ 15. $\dfrac{d}{dx}(\text{Arccos } 3x) =$

(A) $\dfrac{3}{\sqrt{1-x^2}}$ (B) $\dfrac{-1}{3\sin 3x}$ (C) $\dfrac{-3}{\sqrt{1-9x^2}}$ (D) $\dfrac{3}{\sqrt{1-3x^2}}$

(E) $\dfrac{3}{\sqrt{9x^2-1}}$

_____ 16. If $f(x) = 2e^x + e^{2x}$, then $f'''(0) =$

(A) 12 (B) 10 (C) 6 (D) 4 (E) 3

______ 17. Determine the absolute maximum value and the absolute minimum value of the function $f(x) = 2x^3 + 3x^2 - 12x$ over the interval $[-3, 3]$.

(A) Absolute Maximum value is 45; Absolute Minimum value is -7

(B) Absolute Maximum value is 20; Absolute Minimum value is -7

(C) Absolute Maximum value is 3; Absolute Minimum value is 1

(D) Absolute Maximum value is -2; Absolute Minimum value is 1

(E) Absolute Maximum value is 45; Absolute Minimum value is 9

______ 18. In proving that $\lim_{x \to 2} 3x = 6$, what is the largest value of δ corresponding to $\varepsilon > 0$ such that $|3x - 6| < \varepsilon$ whenever $|x-2| < \delta$?

(A) ε (B) $\frac{\varepsilon}{2}$ (C) $\frac{\varepsilon}{3}$ (D) $\frac{\varepsilon}{6}$ (E) 2ε

______ 19. If $y = \ln(\sin x)$, then $\frac{dy}{dx} =$

(A) $\frac{1}{\sin x}$ (B) $\ln(\sin x)$ (C) $\frac{1}{x}\sin x + \ln(\cos x)$

(D) $(\cos x)\ln(\sin x)$ (E) $\cot x$

______ 20. A function f is defined by $f(x) = \begin{cases} \frac{x^2 - 4}{x - 2} & \text{if } x \neq 2 \\ k & \text{if } x = 2 \end{cases}$

If f is continuous at $x = 2$, what is the value of k?

(A) -2 (B) 0 (C) 2 (D) 4 (E) $\frac{1}{2}$

II. Free Response

21. Prove the following derivative formula:

Given: $y = a^{x}$, where a is a constant

Prove: $\dfrac{dy}{dx} =$

________________ 22. Given L feet of fencing, what is the maximum number of square feet that can be enclosed if the fencing is used to make three sides of a rectangular pen, using an existing wall as the fourth side?

________________ 23. Given $f(1) = 2$, $f'(1) = 4$, and $g(x) = \left(f(x)\right)^{-3}$

Determine $\dfrac{d}{dx}\left(g(x)\right)\Big|_{x=1}$

________________ 24. At what points on the graph of $y = \dfrac{4}{x-1}$ is the slope equal to -1 ?

________________ 25. Determine $\dfrac{dy}{dx}$, given that $y\sin x + x\sin y = 1066$

________________ 26. Determine $\dfrac{dy}{dx}$, given that $y = \cos^2 x + \sin^2 x$

________________ 27. Determine $\dfrac{dy}{dx}$, given that $xy + y^2 = 2001$

________________ 28. Determine $\dfrac{dy}{dx}$, given that $y = \dfrac{x^2}{\sin(x)}$

________________ 29. Determine $\dfrac{dy}{dx}$, given that $y = u^2 + 1$ and $u = 3x - 5$

________________ 30. Determine the domain of $y = \sqrt{\dfrac{x}{x-2}}$

________________ 31. Determine the equation of the tangent to the curve $\sin(y) = \cos(x)$ at the point $\left(\dfrac{\pi}{2}, 0\right)$.

________________ 32. A particle projected vertically upward with an initial velocity of 128 ft/sec reaches an elevation $s = 128t - 16t^2$ at the end of *t* seconds. How high does the particle rise?

________________ 33. Is the function *f(x)* continuous at x = 0 ?

(SHOW WORK BELOW)

$$f(x) = \begin{cases} \dfrac{\sin(x)}{x} & \text{when } x \neq 0 \\ 2 & \text{when x = 0} \end{cases}$$

PROVE your answer:

________________ 34. Determine the value of x if $f(x) = x^2$ and $f'(f(x)) + f(f'(x)) = 54$.

________________ 35. If $y = (x+1)^{-2}$, determine the 100^{th} derivative of y with respect to x.

a = ______________ 36. The curve $y = ax^2 + bx + c$ passes through the points $(2, 5)$ and $(-2, -3)$. The value of y is greatest when $x = 1$. Determine the values of a, b, and c.

b = ______________

c= ______________

________________ 37. If the surface area of a sphere is increasing at the rate of 12 square feet per second, how fast is the radius increasing when it is 2 feet?

________________ 38. Solve for y *(to three decimal places)*: $12 = 5^y$

39. Write out the complete definition for continuity.

40. Sketch a curve which satisfies the following conditions:

$$\frac{dy}{dx} > 0 \text{ on } (-\infty, 0) \text{ and } (2, +\infty) \qquad \frac{dy}{dx} < 0 \text{ on } (0, 2)$$

$$\frac{d^2y}{dx^2} > 0 \text{ on } (1, +\infty) \qquad \frac{d^2y}{dx^2} < 0 \text{ on } (-\infty, 1)$$

$$f(0) = 4 \qquad f(2) = 0 \qquad f(1) = 1$$