A. P. Calculus First Semester Examination Mr. Pleacher Name _____

A calculator is NOT allowed on this section of the Exam.

I. Multiple Choice

_____ 1. If $f(x) = e^x$, which of the following is an asymptote to the graph of f?
(A) $y = 0$ (B) $x = 0$ (C) $y = -x$ (D) $y = 1$ (E) $y = x$

_____ 2. If $\log_a\left(2^a\right) = \frac{a}{4}$, then $a =$
(A) 2 (B) 4 (C) 8 (D) 16 (E) 32

_____ 3. The area of a circle is given by $A = \pi r^2$. Assuming that the radius is changing, the formula for the instantaneous rate of change of *A* with respect to *r* is:
(A) π (B) 0 (C) 2π (D) πr^2 (E) $2\pi r$

_____ 4. If $f(x) = 3x^3 - 7x + 9$, then $f''(x) = \frac{d^2y}{dx^2} =$
(A) $9x - 7$ (B) $27x^2 - 7$ (C) $9x^2 - 7$ (D) $9x^2$ (E) $18x$

_____ 5. Let $f(x) = \cos\left(Tan^{-1}x\right)$. What is the range of $f(x)$?
(A) $\frac{-\pi}{2} < y < \frac{\pi}{2}$ (B) $0 < y \le 1$ (C) $0 \le y \le 1$
(D) $-1 \le y \le 1$ (E) $-1 < y < 1$

_____ 6. Evaluate $\lim_{h\to 0}\dfrac{\cos\left(\frac{\pi}{2}+h\right)-\cos\left(\frac{\pi}{2}\right)}{h}$

(A) $-\infty$ (B) -1 (C) 0 (D) 1 (E) ∞

_____ 7. $\log\left(\dfrac{x^2}{3y}\right)$ is equivalent to

(A) $\log(2x)-\log(3y)$ (B) $2\log(x)-3\log(y)$ (C) $2\log(x)-\log(3)+\log(y)$

(D) $\log(x)+\log(2)-\log(3y)$ (E) $2\log(x)-\log(3)-\log(y)$

_____ 8. Evaluate $\log_3\left(\dfrac{1}{27}\right)$

(A) $\dfrac{-1}{3}$ (B) $\dfrac{1}{3}$ (C) -3 (D) 3 (E) 9

_____ 9. The set of all points $\left(e^t, t\right)$ where t is a real number is the graph of:

(A) $y=\dfrac{1}{e^x}$ (B) $y=(e)^{\frac{1}{x}}$ (C) $y=x\,(e)^{\frac{1}{x}}$ (D) $y=\dfrac{1}{\ln x}$ (E) $y=\ln x$

_____ 10. If $y=-x^2+4x+25,$ What is the maximum value for y?

(A) 25 (B) -16 (C) 28 (D) 29 (E) 18

_____ 11. Which of the following is a point of discontinuity for $f(x)=\dfrac{x^2-4}{x^2+2x-3}$?

(A) -3 (B) 2 (C) 0 (D) -1 (E) -2

_____ 12. $\lim\limits_{x\to 0}\left(\left(\dfrac{\sin x}{x}\right)\left(\dfrac{x+1}{x-1}\right)\right)=$

(A) -1 (B) 1 (C) 0 (D) π (E) $+\infty$

_____ 13. The graph of $y=2x^3+5x^2-6x+7$ has a point of inflection at x =

(A) 0 (B) $\dfrac{-3}{5}$ (C) $\dfrac{-5}{6}$ (D) $\dfrac{2}{5}$ (E) None of these

_____ 14. Determine the exact value of $Sin^{-1}\left(\dfrac{\sqrt{3}}{2}\right)$

(A) 0 (B) $\dfrac{\pi}{6}$ radians (C) $\dfrac{1}{2}$ (D) 60 degrees (E) None of these

_____ 15. If $f(x)=\ln x$, then the inverse function $f^{-1}(x)=$

(A) $\dfrac{1}{x}$ (B) $\dfrac{1}{\ln x}$ (C) e^x (D) e^{-x} (E) x

_____ 16. If $f'(x) > 0$ and $f''(x) < 0$ over the same interval, which of the following statements is true for $f(x)$ over that interval?

(A) $f(x)$ is increasing and concave up

(B) $f(x)$ is increasing and concave down

(C) $f(x)$ is decreasing and concave up

(D) $f(x)$ is decreasing and concave down

(E) None of the statements are true

_____ 17. Given a function *f*, *how many* of the following statements are true?

(i) If $f''(a) < 0$, then the graph of $y = f(x)$ is concave upward at $x = a$.

(ii) If $f'(a) < 0$, then the graph of $y = f(x)$ is concave downward at $x = a$.

(iii) If $f'(a) = 0$ and $f''(a) > 0$, then $f(a)$ is a relative maximum.

(iv) If $f'(a) = 0$ and $f''(a) = 0$, then $f'''(a) = 0$.

(A) 0 (B) 1 (C) 2 (D) 3 (E) 4

_____ 18. A square piece of tin has 10 inches on a side. An open box is formed by cutting out equal square pieces from the four corners and then bending up the sides. Determine the length of the side of the squares that will result in the maximum volume of the box.

(A) 1 inch (B) $\frac{3}{5}$ inches (C) $\frac{5}{3}$ inches (D) 5 inches

(E) None of the above

_____ 19. If $f(x) = x^3 - 3x^2 - 2x + 5$ and $g(x) = 2$

Then $g(f(x)) =$

(A) $2x^3 - 6x^2 - 4x + 10$ (B) $2x^2 - 6x + 1$

(C) -6 (D) -3 (E) 2

_____ 20. Which of the following statements is true?

(A) $\log(A - B) = \log\left(\frac{A}{B}\right)$

(B) $\log\left(\frac{A}{B}\right) = \frac{\log A}{\log B}$

(C) $\log\left(\frac{A}{B}\right) = \log(A) - \log(B)$

(D) $\log(A - B) = \log(A) - \log(B)$

II. Free Response

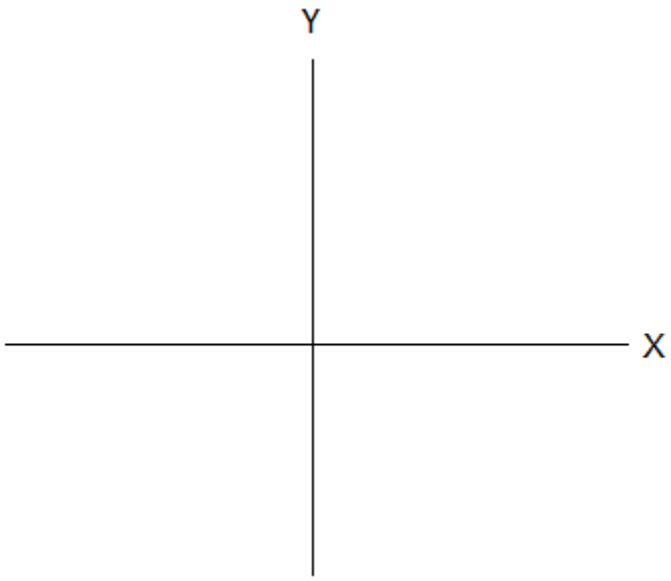
21. Prove the following derivative formula:

Given: $y = \sec(x)$

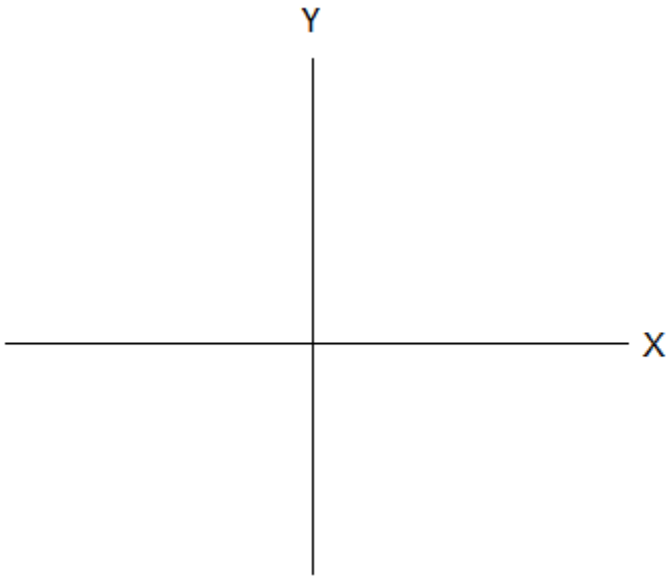
Prove: $\frac{dy}{dx} =$

22. Write out the complete definition of the derivative.

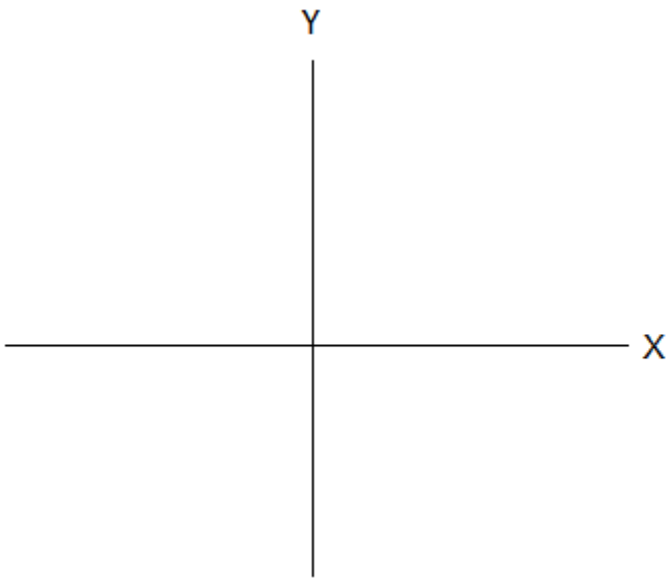
23. Graph the function $y = \dfrac{x - |x|}{2}$ on the axes below:



24. Graph the greatest integer function $y = [x]$ on the axes below:



25. Graph the **derivative** of $y = -2x + 7$ on the axes below:



_____ 26. Evaluate the limit: $\lim_{x \to \infty} \left(\frac{2x^2}{2006 - x^2} \right)$

_____ 27. Evaluate the limit: $\lim_{x \to 3} \frac{2x^2 + x - 9}{x + 3}$

_____ 28. Evaluate the limit: $\lim_{x \to 0} \frac{\sin(x)}{3x}$

_____ 29. Evaluate the limit: $\lim_{x \to 4} \frac{x^2 - 16}{x - 4}$

_____ 30. Evaluate the limit: $\lim_{h \to 0} \frac{\cos\left(2\left(x + h\right)\right) - \cos 2x}{h}$

_____ 31. Determine the differential dy, given that $y = x^4 - 5x^2 + 2006$.

_____ 32. Determine the derivative of $\ln\left(e^{7x}\right)$.

_____ 33. Determine the inverse function of $y = \log_5 x$.

_____ 34. Determine $\frac{d}{dx}\left(x^{\cos x}\right)$

35-36. Given the equation $y = x^3 - 3x^2 + 4$, determine the following:

First Derivative: _____

Increasing on: _____

Decreasing on: _____

Relative Maximum at: _____

Relative Minimum at: _____

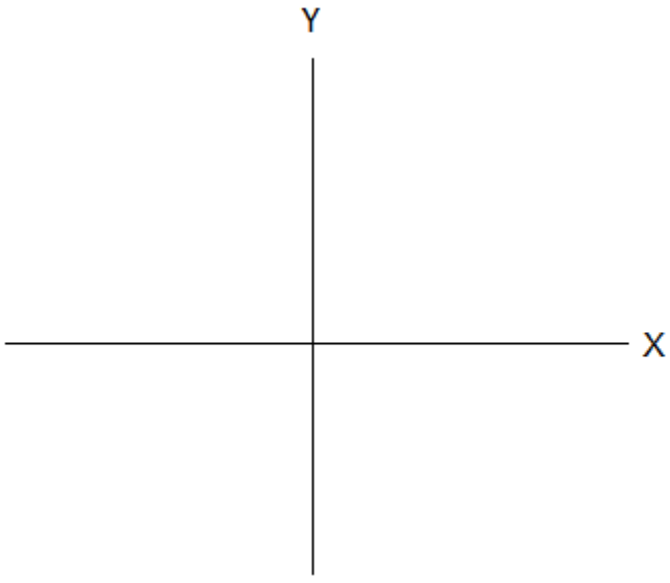
Second Derivative: _____

Concave Up on: _____

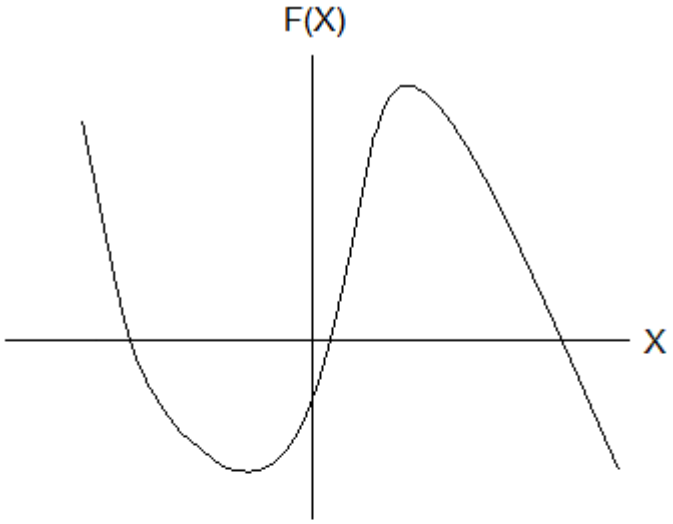
Concave Down on: _____

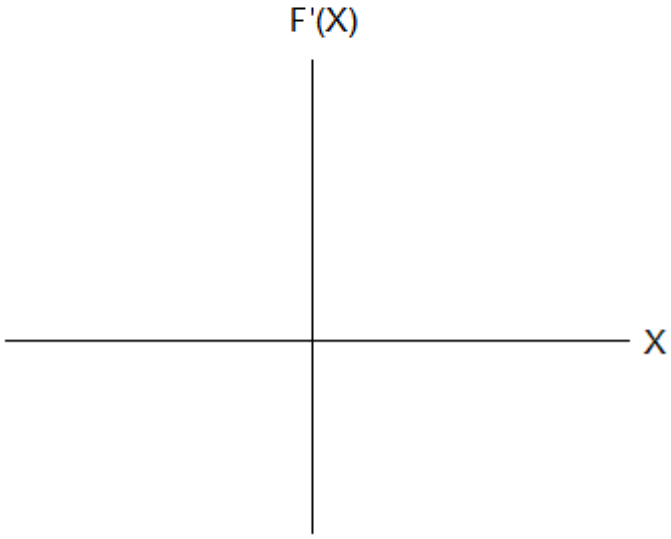
Point of Inflection at: _____

37. Sketch the graph of the curve $f(x) = x^4 - 4x^2$



38. Given the graph of the curve *f(x)*, draw the graph of *f'(x)* on the axes below:



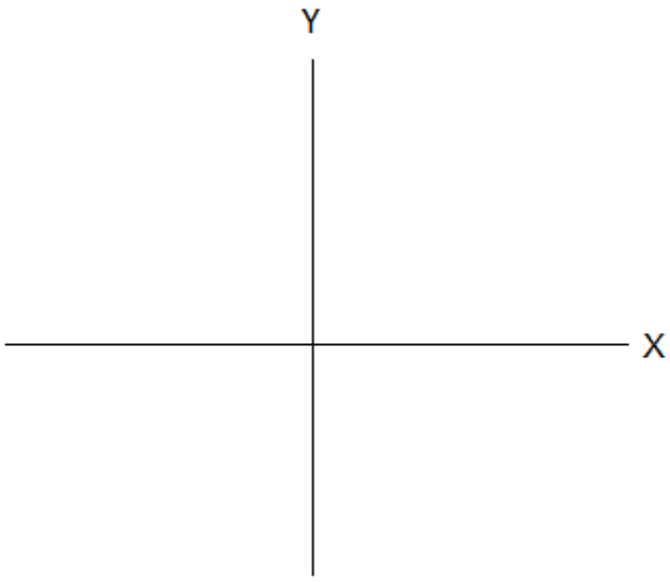


39. Sketch $y = f(x)$, given that

$f(1) = -2$

$f''(x) < 0 \quad for\ x < 1$

$f''(x) > 0 \quad for\ x > 1$



40. Prove the following derivative formula:

Given: $y = \ln(x)$

Prove: $\frac{dy}{dx} =$