

I. Multiple Choice

_____ 1. If $f(x) = \frac{x-1}{x}$ and $g(x) = 1-x$ Then $g(f(x)) =$

(A) $\frac{-x}{1-x}$ (B) $\frac{1}{1-x}$ (C) $\frac{1-x}{x-1}$

(D) $\frac{1}{x}$ (E) $\frac{x}{1-x}$

_____ 2. For $x \neq 4$, the function $h(x)$ is equal to $\frac{x^2 + x - 20}{x - 4}$. What value should be assigned to $h(4)$ to make $h(x)$ continuous at $x = 4$?

(A) 20 (B) 9 (C) 5 (D) 0 (E) -1

_____ 3. In proving that $\lim_{x \rightarrow 10} 2x = 20$, determine the value of δ , given that $\varepsilon = 0.1$.

(A) 0.01 (B) 0.005 (C) 0.05 (D) 0.0025 (E) 0.00005

_____ 4. A ball is dropped from a height of 1 meter. It always bounces to one-half its previous height. The ball will bounce infinitely but it travel a finite distance. What is the distance?

(A) 4 m (B) 3 m (C) $2\frac{31}{32}$ m (D) 2 m (E) It can not be determined

_____ 5. $\lim_{x \rightarrow 6} 4 =$

(A) 4 (B) 6 (C) 0 (D) -4 (E) Does not exist

_____ 6. $\lim_{x \rightarrow 3} \frac{x}{x-3} =$

- (A) 1 (B) 0 (C) $+\infty$ (D) $-\infty$ (E) Does not exist

_____ 7. Determine the points of discontinuity of $f(x) = \frac{3}{x} + \frac{2x}{x+4}$

- (A) $x = 0$ only (B) $x = -4$ only (C) $x = -4, 0$
(D) $x = 0, 4$ (E) No points of discontinuity

_____ 8. $\lim_{x \rightarrow \infty} \frac{x^3 - 2x + 3}{6 - 4x^2 + 3x^3} =$

- (A) $\frac{1}{3}$ (B) $\frac{1}{6}$ (C) $-\frac{1}{3}$ (D) 3 (E) Does not exist

II. Definitions

Write out complete definitions for each of the following on your own paper:

9. Limit

10. Continuity

III. Determine the limits of the following functions, showing all work on your own paper

11. $\lim_{\theta \rightarrow 0} \left(\frac{\sin(8\theta)}{\theta} \right) =$

12. $\lim_{x \rightarrow \infty} \frac{2x^2 - 3}{5 - x^3} =$

13. $\lim_{x \rightarrow -\infty} \frac{2x^4 - 3}{5 - x^3} =$

$$14. \lim_{x \rightarrow 3} (4x - 5) =$$

$$15. \lim_{x \rightarrow -4} \left(\frac{x^3 + 64}{x + 4} \right) =$$

$$16. \lim_{\theta \rightarrow 0} \left(\frac{\sin(7\theta)}{\sin(2\theta)} \right) =$$

$$17. \lim_{x \rightarrow 0} \frac{5 \sin^2 x}{2x^2} =$$

$$18. \lim_{h \rightarrow 0} \frac{(x+h)^2 - x^2}{h} =$$

IV. Miscellaneous

$$19. \text{ Given } f(x) = \begin{cases} 2x - 5, & \text{if } x \leq 2 \\ kx + 3, & \text{if } x > 2 \end{cases}$$

Find the value for the constant k that will make the function continuous at $x = 2$.

20. Is the function $h(x)$ continuous at $x = 0$? **Prove** your answer.

$$h(x) = \begin{cases} \frac{\sin(x)}{x}, & \text{when } x \neq 0 \\ 1, & \text{when } x = 0 \end{cases}$$

21. Using the $\delta - \varepsilon$ definition of the limit, prove that $\lim_{x \rightarrow 3} (5x - 5) = 10$.

V. Graphs

Graph each of the following on your own paper (or graph paper):

22. Draw the graph of a function $h(x)$ whose limit as x approaches 5 does not exist, but $h(5)$ is defined to be 3.

23. Graph $y = |x| - 2$

24. Graph $y = [x]$

25. Graph $f(x) = \begin{cases} -x + 2, & x \leq 1 \\ x - 2, & x > 1 \end{cases}$

Extra Credit (4 points)

26. Determine the limit, showing all work:

$$\lim_{x \rightarrow 2^+} \left(\frac{2 + 5x - 3x^2}{|2 - x|} \right) =$$