Test Chapter 3, 4-1 Derivatives Name _____

I. Multiple Choice

_____ 1. Given the formula for the volume of water in gallons in a cylindrical tank after t minutes

$$V(t) = 20,000\left(1 - \frac{t}{20}\right)^2$$

Find the instantaneous rate of change of V with respect to t at time t = 3 minutes.

(A) 1,700 gallons/minute
(B) -1,700 gallons/minute
(C) 2,200 gallons/minute
(D) –1,800 gallons/minute
(E) 1,800 gallons/minute

_____ 2. Given the parametric equations,

$$x = t^2 \text{ and } y = 2t - 3,$$

find the derivative $\frac{dy}{dx}$.

(A) $\left(\frac{y+3}{2}\right)^2$ (B) $\frac{y+3}{4}$ (C) $\frac{2}{y+3}$

(D) $2\sqrt{x} - 3$ (E) $\frac{1}{2\sqrt{x} - 3}$

_____ 3. Let f be the function defined by $f(x) = 3x^5 - 5x^3 + 2$.

The horizontal tangent lines to the graph of $y = f(x)$ are:

(A) y = 0; y = 3
(B) y = 2; y = 3
(C) y = -1; y = 1
(D) y = 0; y = 2; y = 4
(E) y = 0; y = -1; y = 1

_____ 4. Given $y = \left(x^2 + 5\right)^6$ Determine $\frac{dy}{dx}$

(A) $6\left(2x\right)^5$ (B) $6\left(x^2 + 5\right)^5$

(C) $12x\left(x^2 + 5\right)$ (D) $12x\left(x^2 + 5\right)^5$

_____ 5. The value of the derivative of $f(x) = 3x^2 + 2x + 5$ at $x = -3$ is:

(A) –16 (B) 24 (C) 6x + 2 (D) 14
(E) Nonexistent

_____ 6. $\frac{d}{dx}\left(\frac{x}{2+x}\right) =$

(A) $\frac{1-x}{\left(2+x\right)^2}$ (B) $\frac{2+x-x^2}{\left(2+x\right)^2}$ (C) $\frac{1}{2+x}$

(D) $\frac{2}{\left(2+x\right)^2}$ (E) $\frac{-x}{\left(2+x\right)^2}$

_____ 7 If $f(x) = \pi^2$, then $f'(x) =$

(A) 0 (B) 2π (C) π (D) 1 (E) π^2

_____ 8 If $y = x^6$, $\frac{dy}{dx} =$

(A) $6x^6$ (B) $6x^5$ (C) $5x^5$ (D) $5x^6$ (E) x^5

_____ 9. If $y = \sin^2 x + \cos^2 x$, $\frac{dy}{dx} =$

(A) $2\sin x + 2\cos x$ (B) $2\sin x - 2\cos x$ (C) $4\sin x \cos x$
(D) 1 (E) 0

_____ 10. If $y = \left(x^2 + 1\right)^4$ Then the differential $dy =$

(A) $4\left(x^2 + 1\right)^3$ (B) $4\left(x^2 + 1\right)^3 dx$ (C) $4\left(2x\right)^3 dx$

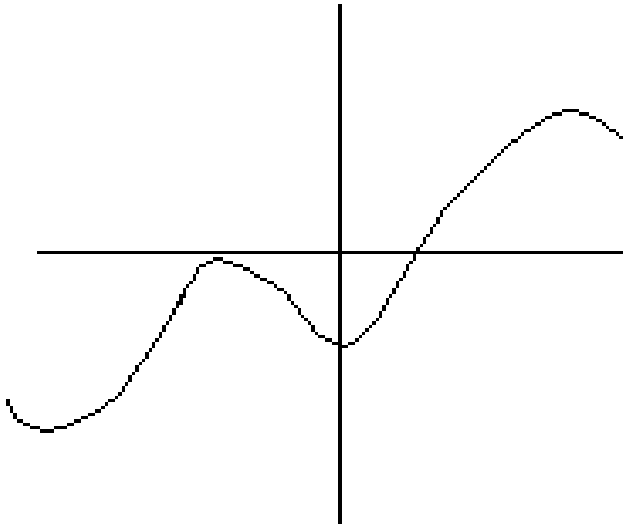
(D) $8x\left(x^2 + 1\right)^3$ (E) $8x\left(x^2 + 1\right)^3 dx$

II. Free Response – DO ON YOUR OWN PAPER

11. Determine the 77^{th} derivative of $y = \sin(3x)$

12. Find $\lim_{h\to 0}\dfrac{(x+h)^3-(x)^3}{h}$

13. Draw a sketch of the derivative of the function given by the following graph:



14. Suppose that the tangent line to $y = g(x)$ at (-3, 5) passes through the point (2, -4).
Find g(-3), g'(-3), and the equation of the tangent line to $y = g(x)$ at (-3, 5).

15. Given $y = \sin^2\left(3x^5\right)$ Determine $\dfrac{dy}{dx}$

16 Given f (1) = 1, f ' (1) = 3, g(1) = -2, and g'(1) = -1.

Determine $\dfrac{d}{dx}\left(f^2(x)-3g\left(x^2\right)\right)\Big|_{x=1}$

17. Given $\frac{1}{y}+\frac{1}{x}=1$ Determine $\frac{dy}{dx}$.

18. Given $\sec(xy)=y$ Determine $\frac{dy}{dx}$.

19. Use differentials to determine the value of $\sqrt[4]{15}$.

20. State and prove the rule for finding either
    (A) the derivative of a quotient or
    (B) the derivative of the sec(x)