Test Chapter 5 Name ____________________

I. Multiple Choice

_____ 1. The largest interval on which $f(x) = x^2 + 4x + 2$ is increasing is

(A) $[0, \infty)$ (B) $[0, -\infty)$ (C) $(-2, \infty)$

(D) $(-\infty, 2)$ (E) $(-\infty, +\infty)$

_____ 2. $f(x) = 2x^2 + 2$ has an absolute minimum on $[-3, 3]$ of

(A) 2 (B) –2 (C) 52 (D) –52 (E) 0

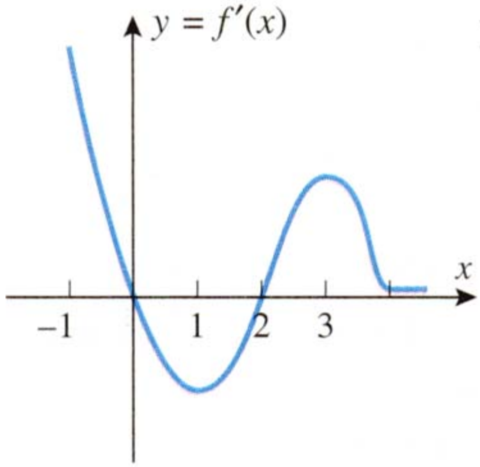
_____ 3. Determine the value of c that satisfies Rolle's Theorem for the function $f(x) = 2x^2 - 8$ on $[-2, 2]$.

(A) 0 (B) –1 (C) 1 (D) 0.5 (E) –0.5

_____ 4. Determine the value of c that satisfies the Mean Value Theorem for $f(x) = x^3$ on $[0, 1]$.

(A) 0 (B) $\frac{\sqrt{3}}{3}$ (C) $\frac{\sqrt{3}}{2}$ (D) $\frac{\sqrt{2}}{2}$ (E) $\frac{\sqrt{2}}{3}$

5-6. Use the graph of $f'(x)$ shown in the figure to answer the questions below.



______ 5. Determine the x-coordinates where $f(x)$ has a relative minima.

(A) $x = 0$ (B) $x = 1$ (C) $x = 0$ and $x = 2$

(D) $x = 2$ (E) $x = 3$

______ 6. Determine the x-coordinates where $f(x)$ has a point of inflection.

(A) $x = 0$ (B) $x = 1$ (C) $x = 0$ and $x = 2$

(D) $x = 2$ (E) $x = 1$ and $x = 3$

______ 7. $f(x) = -x^4 - 6x^2$ is concave up on

(A) $(-\infty, +\infty)$ (B) $(-\infty, -1)$ (C) $(-1, 1)$ (D) nowhere

(E) $(-\sqrt{3}, \sqrt{3})$

______ 8. $f(x) = x^2 - 8x + 7$ has

(A) a relative maximum at x = 4
(B) a relative minimum at x = 4
(C) a relative maximum at x = -4
(D) a relative minimum at x = -4
(E) a relative minimum at x = 0

II. Free Response

9 – 10. Give the following information for the function:

$$y = x^3 - 3x^2 + 4$$

Derivative: ________________________________

Increasing on (___ , ___) and (___ , ___)

Decreasing on (___ , ___)

Relative Maximum at (___ , ___)

Relative Minimum at (___ , ___)

Second Derivative: ________________________

Concave Up on (___ , ___)

Concave Down on (___ , ___)

Points of Inflection at(___ , ___)

11. Sketch a curve that satisfies the following conditions:

$\frac{dy}{dx} > 0$ on $(-\infty, 0)$ and $(2, +\infty)$

$\frac{dy}{dx} < 0$ on $(0, 2)$

$\frac{d^2y}{dx^2} > 0$ on $(1, +\infty)$

$\frac{d^2y}{dx^2} < 0$ on $(-\infty, 1)$

$f(0) = 4$ $f(2) = 0$ $f(1) = 1$

12. Sketch the function y = f(x), given that

$f(1) = 0$

$f'(x) > 0$ for $x < 1$

$f'(x) < 0$ for $x > 1$

13. Sketch the curve with the following properties:

y-axis symmetry
horizontal asymptote: $y = 0$
vertical asymptotes: $x = -2$, $x = 2$
increasing on $(0, 2)$ and $(2, +\infty)$
decreasing on $(-\infty, -2)$ and $(-2,0)$
concave up on $(-2, 2)$
concave down on $(-\infty, -2)$ and $(2, +\infty)$
$f(0) = 2$

14. Determine the constant k so that the function $f(x) = x^2 + \frac{k}{x}$ will have a relative minimum at $x = 2$.

15. The equation $y = x^2 - 3$ has one real root. Approximate it by Newton's method. Let $x_1 = 1$; then determine x_2 and x_3.

16. The height of an object at a given time t is given by the formula $s = s_0 + v_0 t - \frac{1}{2}gt^2$ where s_0 is the initial position, v_0 is the initial velocity, and g is the acceleration due to gravity (use 32 ft/sec^2 for the value of g).

If an object is launched vertically upward from an initial height of 10 feet with an initial speed of 64 ft/sec, use calculus to determine how high the object will rise.

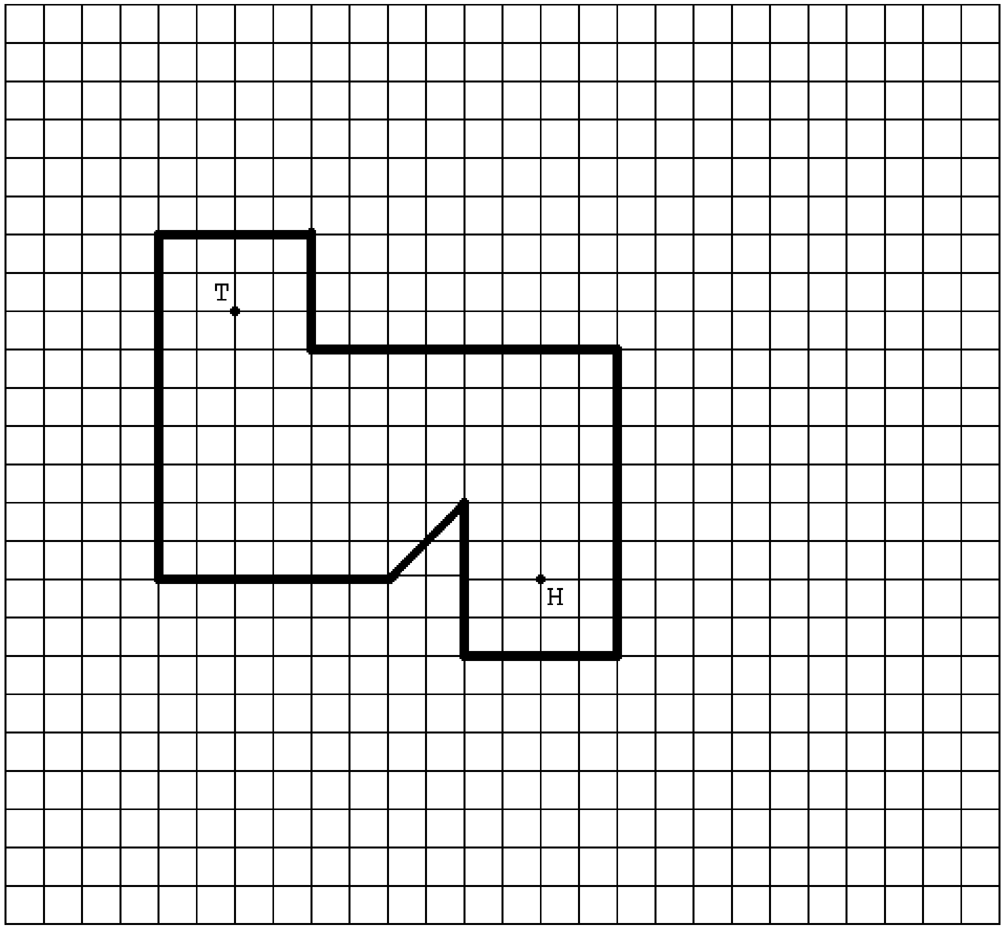
17 - 19. Set up each of the following, but do not solve them. You may stop when you have expressed the quantity you are trying to maximize or minimize in terms of only one variable. When you have finished setting up both problems, go back and **completely solve *one* of them**.

17. A tank with a rectangular base and rectangular sides is to be open at the top. It is to be constructed so that its width is 4 meters and its volume is 36 cubic meters. If building the tank costs $10 per square meter for the base and $5 per square meter for the sides, what is the cost of the least expensive tank?

18. Determine the volume of the largest right circular cone that can be inscribed in a sphere of radius 3 inches.

19. A cylindrical can is to be manufactured to hold 50 cubic inches. Find the height of the can and the radius of the base of the can so that the smallest amount of metal is used in the can.

20. In the diagram below, show how you would hit the golf ball from T so that it would go into the hole at H.


T
H