Test A. P. Calculus Sections 6.1 – 6.6 Name _____

1 – 6. Multiple Choice

_____ 1. $\int dt =$

(A) $t + C$

(B) $x + C$

(C) $tx + C$

(D) $\frac{1}{2}t^2 + C$

(E) $dt + C$

_____ 2. $\int (\sin x - 3\cot x \sin x)\, dx =$

(A) $\cos x + 3\csc x + K$

(B) $\cos x + 3\sin x + K$

(C) $-\cos x - 3\sin x + K$

(D) $-\cos x + 3\csc x + K$

(E) None of the above

_____ 3. $\int x\sqrt{9 - x^2}\, dx =$

(A) $\frac{3}{2}x^2 - \frac{1}{3}x^3 + K$

(B) $\frac{3}{2}x^2 - x^3 + K$

(C) $-\frac{1}{2}\left(\left(9 - x^2\right)^{\frac{3}{2}}\right) + K$

(D) $\dfrac{\left(9 - x^2\right)^{\frac{3}{2}}}{3} + K$

(E) $-\frac{1}{3}\left(\sqrt{9 - x^2}\right)^3 + K$

_____ 4. Suppose that you approximate the area under $g(x) = \sin(x) + 2$ on the domain $\frac{\pi}{2} \le x \le \frac{3\pi}{2}$ with n = 4 subintervals, using right-hand endpoints. The set of x-values you need to use are:

(A) $\left\{ 0, \frac{\pi}{4}, \frac{2\pi}{4}, \frac{3\pi}{4} \right\}$

(B) $\left\{ \frac{\pi}{4}, \frac{2\pi}{4}, \frac{3\pi}{4}, \frac{4\pi}{4} \right\}$

(C) $\left\{ \frac{2\pi}{4}, \frac{3\pi}{4}, \frac{4\pi}{4}, \frac{5\pi}{4} \right\}$

(D) $\left\{ \frac{3\pi}{4}, \frac{4\pi}{4}, \frac{5\pi}{4}, \frac{6\pi}{4} \right\}$

(E) Not enough information was given

_____ 5. $\sum_{k=1}^{4} \left(k^2 + 1 \right) =$

(A) 14

(B) 29

(C) 30

(D) 34

(E) None of the above

_____ 6. If $\frac{dy}{dx} = x\, y^3$ and $y = 1$ when $x = 1$, then determine the value of y when $x = 0$.

Hint: First, solve the differential equation.

(A) $\frac{2}{3}$

(B) $\frac{3}{2}$

(C) $\pm\frac{2}{3}$

(D) $\pm\frac{\sqrt{2}}{2}$

(E) $\pm\sqrt{2}$

7 – 16. Free Response SHOW ALL WORK on your own paper – Do NOT write on the test paper for any problems in this section except for #16.

7. If the points a, b, c, and d are located on the x-axis such that $a < b < c < d$ and if $\int_a^b f(x)\,dx = 5$ and $\int_c^d f(x)\,dx = 8$ and $\int_a^d f(x)\,dx = 9$

Determine the value of $\int_c^b f(x)\,dx$

8. Determine the **exact** area between the curve $y = x^2$ and the x-axis over the interval $0 \le x \le 4$. Use circumscribed rectangles (right-hand endpoints in this problem) and apply a limit.

You will need to apply one of the following:

$$1 + 2 + 3 + \ldots + n = \frac{n(n+1)}{2}$$

$$1^2 + 2^2 + 3^2 + \ldots + n^2 = \frac{n(n+1)(2n+1)}{6}$$

9. Determine the area between the curve $y = x^2 - 1$ and the x-axis from $x = 0$ to $x = 4$.

10. Evaluate $\int_1^3 \frac{(x+2)\,dx}{\sqrt{x^2 + 4x + 7}} =$

11. Evaluate $\int_{-\pi}^{\frac{\pi}{4}} \cos(2x)\,dx =$

12. Evaluate $\int_0^1 \frac{x\,dx}{3x^2 + 1} =$

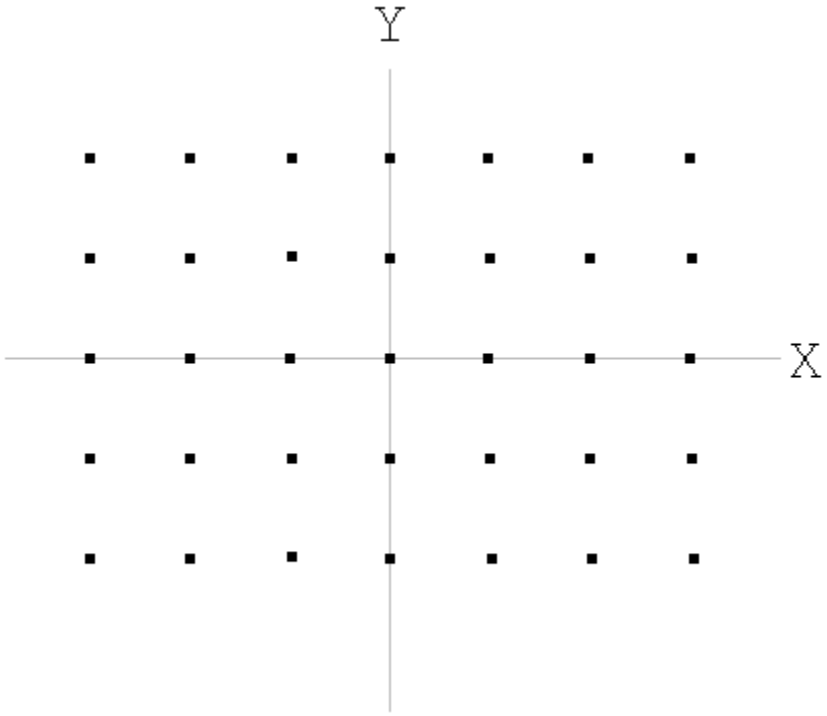
13. Evaluate $\int_1^4 |2x-4|\,dx$ =

You must show ALL WORK, and you must use calculus to solve!

14. Prove by induction: $1+3+5+7+\ldots+(2n-1)=n^2$

15. Solve the differential equation $\frac{dy}{dx}=2+\sin(3x)$, given that $y\left(\frac{\pi}{3}\right)=0$

16. Draw a slope field for the differential equation $\frac{dy}{dx}=-\frac{x}{y}$



Extra Credit (5 points) Evaluate $\prod_{i=2}^{5}(2i-3)=$