

Show all work on your own paper.

Do NOT write anywhere on this test except for question #1.  
You may NOT use a calculator on this test.

1. Give the following information for the function:

$$y = x^4 + 4x^3$$

Derivative: \_\_\_\_\_

Increasing on ( \_\_\_\_ , \_\_\_\_ ) and ( \_\_\_\_ , \_\_\_\_ )

Decreasing on ( \_\_\_\_ , \_\_\_\_ )

Relative Minimum at ( \_\_\_\_ , \_\_\_\_ )

Second Derivative: \_\_\_\_\_

Concave Up on ( \_\_\_\_ , \_\_\_\_ ) and ( \_\_\_\_ , \_\_\_\_ )

Concave Down on ( \_\_\_\_ , \_\_\_\_ )

Points of Inflection at ( \_\_\_\_ , \_\_\_\_ ) and ( \_\_\_\_ , \_\_\_\_ )

2. Sketch the function which is  
 increasing on  $(-\infty, 0)$  and  $(2, +\infty)$ ,  
 decreasing on  $(0, 2)$ ,  
 concave up on  $(1, +\infty)$ ,  
 Concave down on  $(-\infty, 1)$ ,  
 and has a  
 relative maximum at  $(0, 4)$ ,  
 relative minimum at  $(2, 0)$ ,  
 point of inflection at  $(1, 1)$ .

3. Sketch a curve that satisfies the following conditions:

$$\frac{dy}{dx} < 0 \text{ on } (-\infty, 0) \text{ and } (2, +\infty)$$

$$\frac{dy}{dx} > 0 \text{ on } (0, 2)$$

$$\frac{d^2y}{dx^2} < 0 \text{ on } (1, +\infty)$$

$$\frac{d^2y}{dx^2} > 0 \text{ on } (-\infty, 1)$$

$$f(0) = 0$$

$$f(2) = 4$$

$$f(1) = 1$$

4. Sketch  $y = f(x)$ , given that

$$f(1) = -3$$

$$f''(x) > 0 \text{ for } x < 1$$

$$f''(x) < 0 \text{ for } x > 1$$

5 - 7. Sketch the following curves, indicating relative maximum and relative minimum points.

5. Sketch  $y = 6 - 2x - x^2$

6. Sketch  $y = 12 - 12x + x^3$

7. Sketch  $y = -x^4 + 4x^2 + 8$

8. In sketching a curve, how does finding the second derivative help?

9. Find the interval(s) of  $x$  for which the function  $f$  defined by  $f(x) = (x^2 - 3)e^{-x}$  is increasing.

10. Determine the constant  $k$  so that the function

$$f(x) = x^2 + \frac{k}{x} \text{ will have a point of inflection at } x = 1.$$