Test   Curve Sketching   Name ___________________

Show all work on your own paper.
Do NOT write anywhere on this test except for question #1.
You may NOT use a calculator on this test.

1. Give the following information for the function:

$$y = x^3 - 3x^2 + 4$$

Derivative: ____________________________________

Increasing on (___ , ___) and (___ , ___)

Decreasing on (___ , ___)

Relative Maximum at (___ , ___)

Relative Minimum at (___ , ___)

Second Derivative: ____________________________

Concave Up on (___ , ___)

Concave Down on (___ , ___)

Points of Inflection at(___ , ___)

2. Sketch a curve that satisfies the following conditions:

$\frac{dy}{dx} > 0$ on $(-\infty, 0)$ and $(2, +\infty)$ $\frac{dy}{dx} < 0$ on $(0, 2)$

$\frac{d^2y}{dx^2} > 0$ on $(1, +\infty)$ $\frac{d^2y}{dx^2} < 0$ on $(-\infty, 1)$

$f(0) = 4$ $f(2) = 0$ $f(1) = 1$

3. Sketch the function y = f(x), given that

$f(1) = 0$

$f'(x) > 0$ for $x < 1$

$f'(x) < 0$ for $x > 1$

4. Sketch the curve with the following properties:
   y-axis symmetry
   horizontal asymptote: $y = 0$
   vertical asymptotes: $x = -2$, $x = 2$
   increasing on $(0, 2)$ and $(2, +\infty)$
   decreasing on $(-\infty, -2)$ and $(-2, 0)$
   concave up on $(-2, 2)$
   concave down on $(-\infty, -2)$ and $(2, +\infty)$
   $f(0) = 2$

5 – 7. Sketch the following curves, indicating relative maximum and relative minimum points.

5. Sketch $y = x^4 - 2x^2 + 5$

6. Sketch $y = x^4 + 4x^3$

7. Sketch $y = 12 - 12x + x^3$

8. In sketching a curve, how does finding the first derivative help?

9. Find the interval(s) of *x* for which the function *f* defined by $f(x) = \left(x^2 - 3\right)e^{-x}$ is decreasing.

10. Determine the constant *k* so that the function $f(x) = x^2 + \dfrac{k}{x}$ will have a relative minimum at $x = 2$.