Summary of integration Techniques

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I. Check the Basic Formulas

1. $\int du = u + C$
2. $\int k\, du = k\int du$
3. $\int (du + dv) = \int du + \int dv$
4. $\int u^n du = \frac{u^{n+1}}{n+1} + C, \quad n \neq -1$
5. $\int \frac{du}{u} = \ln|u| + C$
6. $\int e^u du = e^u + C$
7. $\int a^u du = \frac{a^u}{\ln a} + C$
8. $\int \sin u\ du = -\cos u + C$
9. $\int \cos u\ du = \sin u + C$
10. $\int \sec^2 u\ du = \tan u + C$
11. $\int \csc^2 u\ du = -\cot u + C$
12. $\int \sec u\ \tan u\ du = \sec u + C$
13. $\int \csc u \cot u\ du = -\csc u + C$
14. $\int \frac{du}{\sqrt{1-u^2}} = Sin^{-1}u + C$ (or $= -Cos^{-1}u + C$)
15. $\int \frac{du}{1+u^2} = Tan^{-1}u + C$ (or $= -Cot^{-1}u + C$)
16. $\int \frac{du}{u\sqrt{u^2-1}} = Sec^{-1}u + C$ (or $= -Csc^{-1}u + C$)

II. Try Substitutions

A. Let u = complex algebraic expression. Then determine du

B. Try trigonometric substitutions

1. For powers of sin, cos, tan, cot, sec, and csc

a. If sin or cos is raised to an even positive power,

use $\sin^2\theta = \dfrac{1-\cos 2\theta}{2}$ or $\cos^2\theta = \dfrac{1+\cos 2\theta}{2}$

b. If sin or cos is raised to an odd power,

use $\sin^2 + \cos^2 = 1$

c. If sin or cos is raised to a negative even power,

use $\sin^2\theta + \cos^2\theta = 1$ and either

$\sin^2\theta = \dfrac{1}{\csc^2\theta}$ or $\cos^2\theta = \dfrac{1}{\sec^2\theta}$

d. If tan or cot is raised to any power,

use $\tan^2\theta = \sec^2\theta - 1$ or $\csc^2\theta = \cot^2\theta + 1$

1. if tan or cot reduces to a power of 1,

use $\tan\theta = \dfrac{\sin\theta}{\cos\theta}$ or $\cot\theta = \dfrac{\cos\theta}{\sin\theta}$

e. If sec or csc is raised to an even power,

use $\tan^2\theta = \sec^2\theta - 1$ or $\csc^2\theta = \cot^2\theta + 1$

f. If the sec or csc is raised to a power of 1,

multiply by $\dfrac{\sec\theta + \tan\theta}{\sec\theta + \tan\theta}$ or $\dfrac{\csc\theta + \cot\theta}{\csc\theta + \cot\theta}$

2. For integrals involving $a^2 - u^2$, $u^2 - a^2$, or $a^2 + u^2$,

a. If $a^2 - u^2$, let $u = a\sin\theta$

b. If $a^2 + u^2$, let $u = a\tan\theta$

c. If $u^2 - a^2$, let $u = a\sec\theta$

d. In particular cases, these result in the following:

$$\int \frac{du}{a^2+u^2} = \frac{1}{a}\tan^{-1}\left(\frac{u}{a}\right) + C$$

$$\int \frac{du}{\sqrt{a^2-u^2}} = \sin^{-1}\left(\frac{u}{a}\right) + C$$

$$\int \frac{du}{\sqrt{u^2-a^2}} = \frac{1}{a}\ln\left|u + \sqrt{u^2-a^2}\right| + C$$

$$\int \frac{du}{\sqrt{u^2+a^2}} = \frac{1}{a}\ln\left|a + \sqrt{u^2+a^2}\right| + C$$

C. Complete the square for integrals involving $ax^2 + bx + c$

D. Use Partial Fractions if the integral involves a fraction in which the degree of the numerator is less than the degree of the denominator

1. If the denominator can be factored into linear factors, use the Heaviside Method:

$$\frac{f(x)}{g(x)} = \frac{A}{x - r_1} + \frac{B}{x - r_2} + \frac{C}{x - r_3} + \ldots$$

2. If the denominator can be factored into quadratic factors,

$$\frac{f(x)}{g(x)} = \frac{Ax + B}{r_1 x^2 - t_1} + \frac{C}{x - t_2} + \ldots$$

3. If the denominator factors into a binomial to a power n,

$$\frac{f(x)}{g(x)} = \frac{A}{x - r_1} + \frac{B}{(x - r_1)^2} + \ldots + \frac{Q}{(x - r_1)^n}$$

E. If the integral involves a fraction in which the degree of the numerator is greater than or equal to the degree of the denominator, divide the numerator by the denominator.

III. Try Integration by Parts

$$\int u\, dv = uv - \int v\, du$$