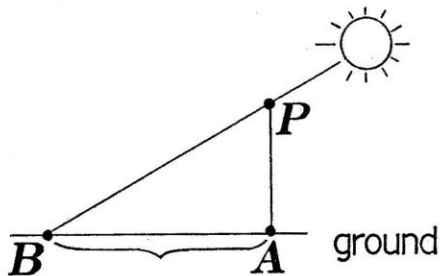


Story about Eratosthenes

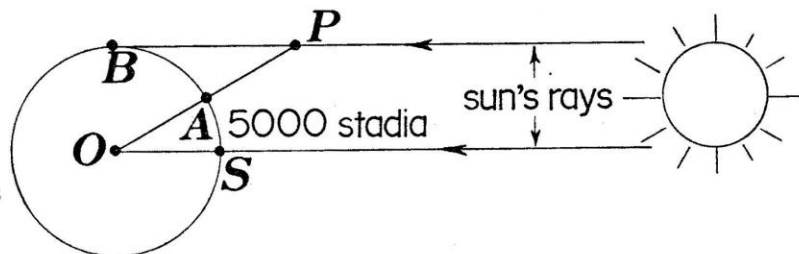
Eratosthenes, who lived in the third century B.C., was a friend of Archimedes. Both of them liked to work with numbers, and together they knew, even at that early date, that the earth was round.

Eratosthenes decided to find the distance around the earth (the earth's *circumference*). It was known that on 21 June the noon sun was directly overhead at Syene. (The town is now called Aswan. You can find it on a map of Egypt.) Alexandria was 5000 *stadia* north of Syene. (Stadia were the units of length used by Egyptians for long distances.) Eratosthenes found that on 21 June a tall vertical pole at Alexandria cast a shadow on the ground. (See the diagram: A represents Alexandria, AP is the pole, and AB is the shadow.) He was able to measure $\angle BPA$ and found it to be about $1/50$ of a complete circle.

Next he decided to assume that the sun's rays coming into Syene and Alexandria on 21 June were parallel. Then he made a picture like this, where S is Syene and O is the earth's center.



Since \overline{PB} and \overline{OS} are parallel, $\angle BPA = \angle AOS$. Since $\angle BPA$ was about $1/50$ of a circle, $\angle AOS$ was also $1/50$ of a circle. Then he knew that the arc \widehat{AS} , which was 500 stadia long, was also $1/50$ of the entire circumference of the earth. He could find the circumference of the earth by multiplying 5000 stadia by 50. This gave him 250 000 stadia.



A later writer gives information leading us to think that 250 000 stadia is about 40 230 kilometers, which is close to the modern measurement of the earth's circumference.

Problem

Find the modern measurement of the earth's equator if it is known that the earth's diameter at the equator is about 12 800 kilometers. (The result should be close to the circumference found by Eratosthenes.)