Converse, Inverse, and Contrapositive

-- Excerpts from *Geometry* by Harold Jacobs

Lewis Carroll, the author of *Alice's Adventures in Wonderland and Through the Looking Glass*, was a mathematics teacher and wrote stories as a hobby. Consider the following conversation at the Mad Hatter's Tea Party:

"Then you should say what you mean," the March Hare went on.
"I do," Alice hastily replied, "at least -- at least I mean what I say -- That's the same thing, you know."
"Not the same thing a bit!" said the Hatter. "Why, you might just as well say that 'I see what I eat' is the same as "I eat what I see'!"
"You might just as well say," added the March Hare, "that 'I like what I get' is the same thing as 'I get what I like'!"
"You might just as well say," added the Dormouse, who seemed to be talking in his sleep, "that 'I breathe when I sleep' is the same thing as 'I sleep when I breathe'!"
"It is the same thing with you," said the Hatter, and here the conversation dropped, and the party sat silent for a minute.

Carroll is playing with pairs of related statements called converses. The hatter, the Hare, and the Dormouse are right: the sentences in each pair do not say the same thing at all. Consider the Dormouse's example. If we change his two statements into "if-then" form, we get:

If I am sleeping, then I am breathing.
and
If I am breathing, then I am sleeping.

Although both statements may be true of the Dormouse, the first statement is true and the second statement is false for most human beings.

Notice that the hypothesis and the conclusion are interchanged, and this is called the converse. There are also two other statements that can be formed from the conditional -- the inverse and the contrapositive -- and they are summarized below:

Conditional statement:	If a, then b.
Converse:	If b, then a.
Inverse:	If not a, then not b.
Contrapositive:	If not b, then not a.

Following each of the numbered statements below are three lettered statements. Identify the relationship of each of the lettered statements to the numbered statement if possible. Write "converse," "inverse," "contrapositive," "original statement," or "none." as appropriate.

1. If you live in Atlantis, then you need a snokel.
   a) If you do not live in Atlantis, then you do not need a snorkel.
   b) If you need a snorkel, then you live in Atlantis.
   c) If you do not need a snorkel, then you do not live in Atlantis.

2. If you are over ninety, the Chop Chop Studio will give you free karate lessons.
   a) If the Chop Chop Studio won't give you free karate lessons, then you aren't over ninety.
   b) If you are ninety or less, the Chop Chop Studio will not give you free karate lessons.
   c) The Chop Chop Studio will give you free karate lessons if you are over ninety.

3. All Eskimos like pie.
   a) If someone likes pie, he is an Eskimo.
   b) If someone is not an Eskimo, then he likes pie.
   c) A person who does not like pie is not an Eskimo.

4. Lady kangaroos do not need handbags.
   a) If a kangaroo is not a lady, then it needs a handbag.
   b) If it needs a handbag, then it is not a lady kangaroo.
   c) A kangaroo does not need a handbag if it is a lady.

Write the indicated statement for each of the following sentences:

5. If the moon is full, the vampires are out. *Converse*

6. If a giraffe has a sore throat, then gargling doesn't help much. *Contrapositive*

7. If we have been receiving signals from Jupiter, it may not be wise to go there. *Inverse*

8. You cannot comprehend geometry if you do not know how to reason deductively. *Converse*

The advertising slogan, "When you're out of Schlitz, you're out of beer" is a rather unusual one.

9. Is it necessarily true?
10. Write its converse, inverse, and contrapositive.
11. Which of these statements are true and which are false?