

Indirect Proof – Sherlock Holmes

Here is the beginning of "The Adventure of the Dancing Men" by Sir Arthur Conan Doyle, one of his Sherlock Holmes stories:

Holmes had been seated for some hours in silence with his long, thin back curved over a chemical vessel in which he was brewing a particularly malodorous product. His head was sunk upon his breast, and he looked from my point of view like a strange, lank bird, with dull grey plumage and a black top-knot.

"So, Watson," said he, suddenly, "you do not propose to invest in South African securities?" I gave a start of astonishment. Accustomed as I was to Holmes's curious faculties, this sudden intrusion into my most intimate thoughts was utterly inexplicable.

"How on earth do you know that?" I asked.

He wheeled round upon his stool, with a steaming test-tube in his hand and a gleam of amusement in his deep-set eyes.

"Now, Watson, confess yourself utterly taken aback," said he.

"I am."

"I ought to make you sign a paper to that effect."

"Why?"

"Because in five minutes you will say that it is all so absurdly simple."

"I am sure that I shall say nothing of the kind."

"You see, my dear Watson" — he propped his test-tube in the rack and began to lecture with the air of a professor addressing his class — "it is not really difficult to construct a series of inferences, each dependent upon its predecessor and each simple in itself. If, after doing so, one simply knocks out all the central inferences and presents one's audience with the starting-point and the conclusion, one may produce a startling, though possibly a meretricious, effect. Now, it was not really difficult, by an inspection of the groove between your left forefinger and thumb, to feel sure that you did not propose to invest your small capital in the goldfields."

"I see no connection."

"Very likely not; but I can quickly show you a close connection. Here are the missing links of the very simple chain:

1. You had chalk between your left finger and thumb when you returned from the club last night.
2. You put chalk there when you play billiards to steady the cue.
3. You never play billiards except with Thurston.
4. You told me four weeks ago that Thurston had an option on some South African property which would expire in a month, and which he desired you to share with him.
5. Your cheque-book is locked in my drawer, and you have not asked for the key.
6. You do not propose to invest your money in this manner."

"How absurdly simple!" I cried.

"Quite so!" said he, a little nettled. "Every problem becomes very childish when once it is explained to you."

Holmes' reference to "a series of inferences, each dependent upon its predecessor" is a description of the form of a simple direct proof. He also says, "after doing so, one simply knocks out all the central inferences and presents one's audience with the starting-point and the conclusion." The starting-point and conclusion constitute the theorem being proved.

The theorem that Holmes is proving to Watson is
"Since there is chalk between your finger and thumb, you are not going to invest in the gold fields."

The proof is:

1. You had chalk between your left finger and thumb when you returned from the club last night.
2. You put chalk there when you play billiards to steady the cue.
3. You only play billiards with Thurston.
4. You told me four weeks ago that Thurston had an option on some South African property which would expire in a month, and which he desired you to share with him.
5. Your cheque-book is locked in my drawer, and you have not asked for the key.

Here is where the indirect proof begins (by assuming the opposite of what you want to prove and then showing a contradiction):

6. Suppose you wanted to invest in this property.
7. Then you would have needed your checkbook, and you would have asked me for the key to my drawer.
8. But you did not ask me for the key (step 5) -- this is the contradiction.
9. Therefore, you do not intend to invest your money in the gold fields of South Africa.