The Science and Math Connection

Chemistry and trigonometry provide a bonding experience

STUDENTS LEARN MOST EFFECTIVELY if they are able to apply inquiry and problem-solving skills to problems that emphasize practical applications. Many experts stress connecting science to other disciplines, such as mathematics, and modeling word problems to real-world situations.

Making a connection between mathematics and chemistry (determining the optimal angle between the atoms of covalent bonds) should help answer the trigonometry student's question, "Why do we need to learn these identities and when will we ever use them?" Several trigonometric identities are necessary when doing the proof of the optimal angle for a molecule with four identical atoms bonded to a central atom that has a complete valence shell. This lesson could be taught collaboratively by a chemistry and a mathematics teacher. Manipulatives should be used to aid students' understanding of the lesson.

BASIC INFORMATION

Atoms form covalent bonds with other atoms to create molecules. A covalent bond is formed when two atoms share a pair of electrons. The number of covalent bonds that an atom can form depends on the number of available electrons found in its outermost (valence) shell. In a single covalent bond, the sharing of a pair of electrons forms the bond that holds two atoms together. However, when considering a polyatomic molecule (a molecule in which there are two or more atoms bonded to a central atom) it is important to realize that there are interactions that occur between the covalent bonds that determine the three-dimensional shape of the molecule.

BY MICHAEL PLEACHER

What are these interactions that occur between covalent bonds? An electron is by definition a negatively charged atomic particle. In a polyatomic molecule, there are two or more covalent bonds. Because each bond is composed of negatively charged electrons, the negative charges found on the electrons that compose the bonds repel each other. Ultimately, the molecule will be arranged in three dimensions such that the repulsion between the electron pairs of different bonds is at a minimum. The repulsive forces between the electron pairs of different covalent bonds causes the bonds to remain as far apart as possible. The valence-shell electron-pair repulsion (VSEPR) model is used by scientists to account for the geometric arrangements of covalent bonds around a central atom that minimize the repulsion between the electron pairs of the covalent bonds.

The simplest molecular shape that can be explained by the VSEPR model is that of a molecule in which two atoms are bonded covalently to a central atom to com-

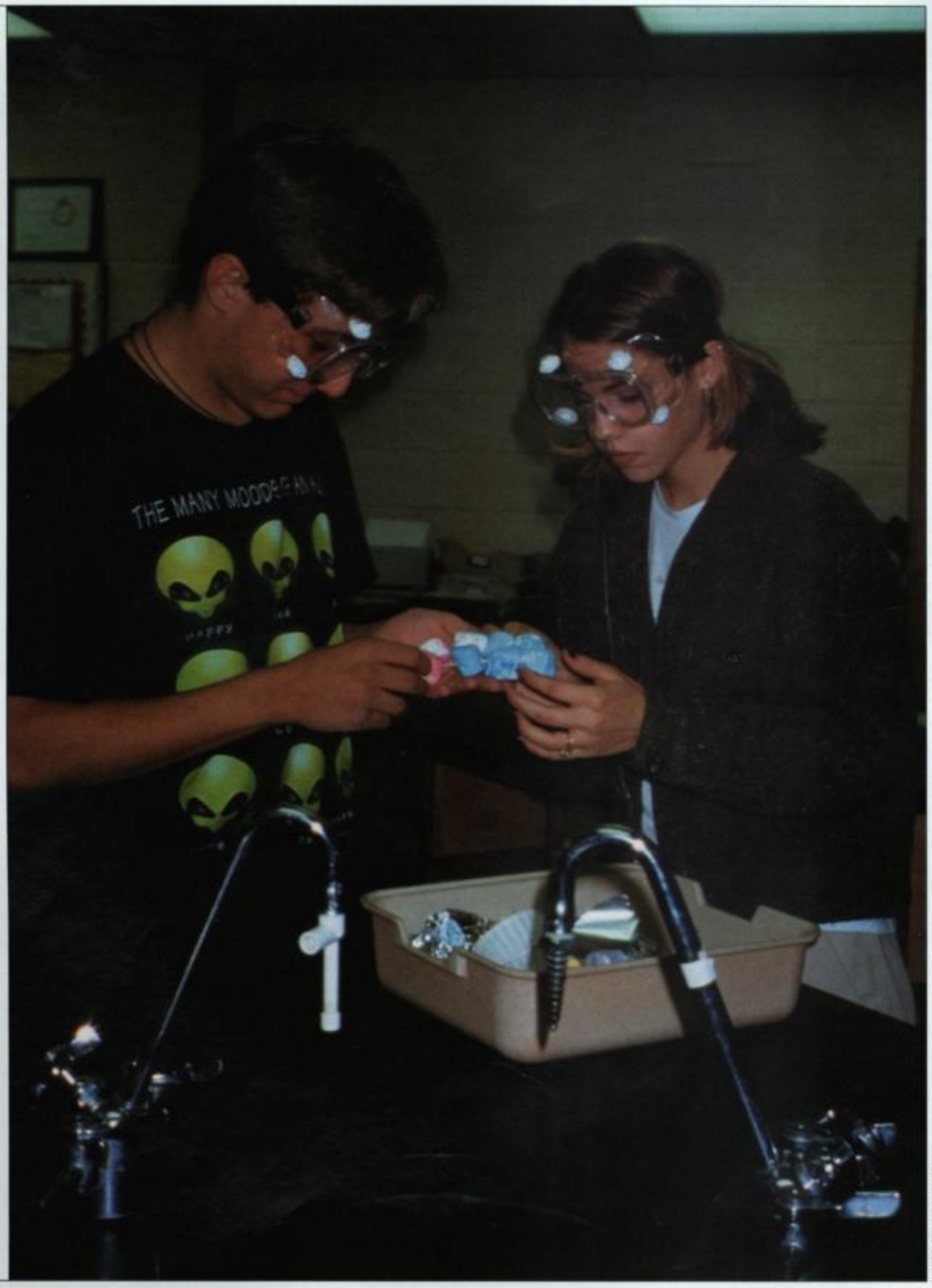


PHOTO BY LAUREN BEBEN

plete its valence shell. Carbon dioxide (with the molecular formula CO_2) is an example of a molecule in which two atoms are bonded covalently to the central atom (C), leaving no nonbonding pairs of electrons.

A Lewis structure is a two-dimensional representation of a molecule's structure. The Lewis structure for CO_2 appears in Figure 1. The electron pairs that create the covalent bonds between the carbon atom and the oxygen atoms repel each other. In order to minimize the repulsion between the covalent bonds, the bonds must be separated from each other by 180°. In this case, the Lewis structure accurately describes both the two-dimensional and three-dimensional shape of the molecule. A polyatomic molecule that is composed of two atoms covalently bonded to a central atom (leaving no non-bonding pairs of electrons) takes on a linear conformation and a characteristic bond angle of 180°.

Knowledge of mathematics means much more than just memorizing information or facts; it requires the ability to use information to reason, think, and solve problems.

Using the VSEPR model, students can examine the geometry of a molecule that is composed of three identical atoms covalently bonded to a central atom, leaving no nonbonding electrons. Boron trifluoride (BF_3) is a molecule that fits this description. The Lewis structure of BF_3 that appears in Figure 1 accounts for molecular shape in only two dimensions. In reality, the molecule exists in three dimensions. The two-dimensional molecular model in this figure suggests that the optimal bond angle for BF_3 is 120°, and that all four atoms of the molecule are in the same plane.

Is there a three-dimensional conformation that would result in a greater bond angle and thus a greater distance between the bonds? Intuitively, the answer is no. If boron were moved out of the plane, the angle in question (FBF) would become smaller, less than 120°. When this angle is reduced, the distance between the two fluorine atoms of the angle is reduced as well. If the two fluorine atoms move closer to each other, the electrons that form the bonds are also brought closer together. If these electrons are brought closer together, they will experience more repulsion. The three-dimensional conformation that BF_3 must take on in order to minimize the repulsion between the covalent bonds is a trigonal planar conformation with an optimal bond angle of 120°.

POLYATOMIC MODELING

A more interesting problem of molecular geometry is encountered when dealing with a molecule comprised of four atoms covalently bonded to a central atom leaving no nonbonding electron pairs. A common example of such a molecule is methane (CH_4). The Lewis structure for CH_4 also appears in Figure 1. The Lewis structure suggests that the optimal bond angle for methane is 90°. Does a three-dimensional conformation exist for methane that would allow bond angles greater than 90°? If such a conformation exists, the hydrogen atoms would be farther apart from each other. How does one go about finding the optimal bond angle that places these four hydrogen atoms at points in space that are the greatest distance from each other?

THREE-DIMENSIONAL MODELS

Students should be encouraged to experiment with

FIGURE 1.

Lewis structures for CO_2, BF_3, and CH_4.

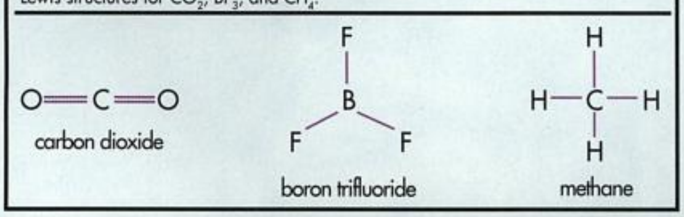
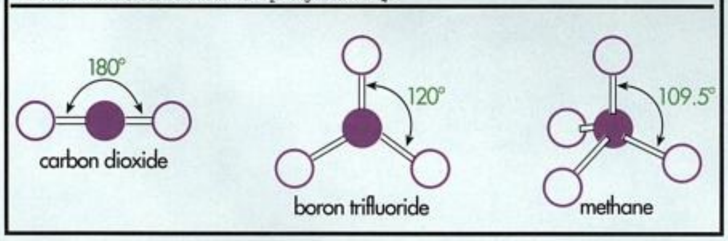


FIGURE 2.

Three-dimensional models of CO_2, BF_3, and CH_4.



manipulatives, such as gum drops and toothpicks or straws and marshmallows, to build the three-dimensional models of carbon dioxide, boron trifluoride, and methane (Figure 2). The three-dimensional model of methane is a tetrahedron, with the carbon atom at the center of the tetrahedron and the four hydrogen atoms at the vertices (Figure 3). Students should be encouraged to construct cardboard or paper triangles for Figures 4, 5, and 6 and to use a protractor to measure the bond angles of each of their models. This will help students to visualize the methane model and understand the following discussion.

To determine the optimal bond angle, draw a perpendicular line from the carbon atom (C) to the plane containing three of the hydrogen atoms. Let Q represent the foot of this perpendicular line (Figures 3 and 4), and let y represent the distance between the carbon atom and any of the hydrogen atoms. Let a represent the distance from Q to one of the hydrogen atoms, and let x represent the measure of the required bond angle.

In Figure 3, note that Q is the circumcenter of the equilateral triangle formed by the three hydrogen atoms that lie in the bottom plane. Because the ΔHHH is equilateral, each of the angles HQH measures 120°. In Figure 4, mHCQ = $(180-x)^\circ$ and mCHQ = $(x - 90)^\circ$. It follows that $\cos(x-90)^\circ = a / y$ or $a = y \cos(x-90)^\circ$. Using the trigonometric identities $\cos(-\theta) = \cos(\theta)$ and $\cos(90 - \theta)^\circ = \sin(\theta)$, it follows that $a = y \cos(90 - x)^\circ$ and that $a = y \sin(x)$.

Now, examine the triangle formed by two hydrogen atoms and point Q (Figure 5). The altitude from point Q divides the triangle HQH into two congruent triangles HQT and HQT (hypotenuse - leg theorem). So, the vertex angle HQH is divided into two angles whose measures are each 60°.

Figure 6 represents the triangle formed by the carbon atom and two hydrogen atoms. Using the definition of sine, it can be shown that $\sin(x/2) = (a\sqrt{3})/(2y)$, where x is the required bond angle. Now substituting the half angle identity, $\sin(x/2) = +/- \sqrt{(1-\cos x)/(2)}$ and the result from above, $a = y \sin(x)$, the following equation is obtained: $+/- \sqrt{(1-\cos x)/2} = (\sqrt{3} \sin x)/2$. Squaring both sides, one obtains the following result: $(1 - \cos x)/2 = 3\sin^2 x/4$.

Using the identity $\sin^2 x = 1 - \cos^2 x$, it can be shown that $3\cos^2(x) - 2\cos(x) - 1 = 0$ from which $\cos(x) = -.33$ or $\cos(x) = 1$. Finally, the value of x is 109.5°, which is the measure of the required bond angle.

CONNECTING MATH TO SCIENCE

It is important for students to make connections between mathematics and other disciplines. Knowledge of mathematics means much more than just memorizing information or facts; it requires the ability to use information to reason, think, and solve problems. By themselves, trigonometric identities are just facts, but applying them to a real-world problem will give students a deeper appreciation of those identities and of mathematics. This manipulation of several trigonometric identities allows students to discover for themselves that the optimal bond angle for methane is 109.5°, not 90° as suggested by the 2-dimensional representation. Hopefully, students will begin to value and use the connections between mathematics and other disciplines. ◆

Michael Pleacher is a medical student at Medical College of Virginia, 2715 East Grace Street, Rear Apt., Richmond, VA 23223; e-mail address: pleacherm@cbil1.cbil.vcu.edu.

FIGURE 3.

Methane model.

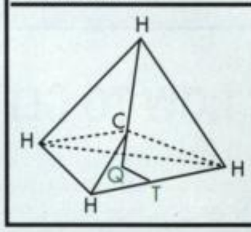


FIGURE 4.

Tetrahedron formed by two hydrogen atoms, the carbon atom, and point Q.

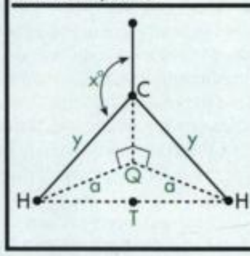


FIGURE 5.

Triangle formed by two hydrogen atoms and point Q.

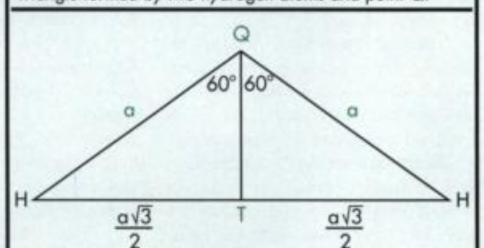


FIGURE 6.

Triangle formed with carbon and two hydrogen atoms as vertices.