Trigonometric Identities

A List by Mr. Pleacher

I. Reciprocal Identities

1. $\sin\theta = \dfrac{1}{\csc\theta}$
2. $\cos\theta = \dfrac{1}{\sec\theta}$
3. $\tan\theta = \dfrac{1}{\cot\theta}$
4. $\cot\theta = \dfrac{1}{\tan\theta}$
5. $\sec\theta = \dfrac{1}{\cos\theta}$
6. $\csc\theta = \dfrac{1}{\sin\theta}$

II. Quotient Identities

7. $\tan\theta = \dfrac{\sin\theta}{\cos\theta}$
8. $\cot\theta = \dfrac{\cos\theta}{\sin\theta}$

III. Pythagorean Identities

9. $\sin^2\theta + \cos^2\theta = 1$ $\quad \sin^2\theta = 1 - \cos^2\theta$ $\quad \cos^2\theta = 1 - \sin^2\theta$
10. $\tan^2\theta + 1 = \sec^2\theta$ $\quad \tan^2\theta = \sec^2\theta - 1$ $\quad \sec^2\theta - \tan^2\theta = 1$
11. $1 + \cot^2\theta = \csc^2\theta$ $\quad \cot^2\theta = \csc^2\theta - 1$ $\quad \csc^2\theta - \cot^2\theta = 1$

IV. Cofunction Identities

12. $\sin\left(\dfrac{\pi}{2} - \theta\right) = \cos\theta$
13. $\cos\left(\dfrac{\pi}{2} - \theta\right) = \sin\theta$
14. $\tan\left(\dfrac{\pi}{2} - \theta\right) = \cot\theta$
15. $\cot\left(\dfrac{\pi}{2} - \theta\right) = \tan\theta$
16. $\sec\left(\dfrac{\pi}{2} - \theta\right) = \csc\theta$
17. $\csc\left(\dfrac{\pi}{2} - \theta\right) = \sec\theta$

V. Even/Odd Identities

18. $\sin(-\theta) = -\sin\theta$
19. $\cos(-\theta) = \cos\theta$
20. $\tan(-\theta) = -\tan\theta$
21. $\csc(-\theta) = -\csc\theta$
22. $\sec(-\theta) = \sec\theta$
23. $\cot(-\theta) = -\cot\theta$

VI. Sum and Difference Identities

24. $\sin(A+B) = \sin A\cos B + \cos A\sin B$
25. $\sin(A-B) = \sin A\cos B - \cos A\sin B$
26. $\cos(A+B) = \cos A\cos B - \sin A\sin B$
27. $\cos(A-B) = \cos A\cos B + \sin A\sin B$
28. $\tan(A+B) = \dfrac{\tan A + \tan B}{1 - \tan A\tan B}$
29. $\tan(A-B) = \dfrac{\tan A - \tan B}{1 + \tan A\tan B}$

VII. Double Angle Identities

30. $\sin 2A = 2\sin A\cos A$
31. $\cos 2A = \cos^2 A - \sin^2 A$ $\qquad \cos 2A = 2\cos^2 A - 1$ $\qquad \cos 2A = 1 - 2\sin^2 A$
32. $\tan 2A = \dfrac{2\tan A}{1 - \tan^2 A}$

VIII. Half Angle Identities

33. $\sin\left(\dfrac{\theta}{2}\right) = \pm\sqrt{\dfrac{1-\cos\theta}{2}}$ $\qquad \sin^2\theta = \dfrac{1-\cos 2\theta}{2}$

34. $\cos\left(\dfrac{\theta}{2}\right) = \pm\sqrt{\dfrac{1+\cos\theta}{2}}$ $\qquad \cos^2\theta = \dfrac{1+\cos 2\theta}{2}$

35. $\tan\left(\dfrac{\theta}{2}\right) = \pm\sqrt{\dfrac{1-\cos\theta}{1+\cos\theta}}$ $\qquad \tan\left(\dfrac{\theta}{2}\right) = \dfrac{1-\cos\theta}{\sin\theta} = \dfrac{\sin\theta}{1+\cos\theta}$

IX. Product-to-Sum Identities

36. $\sin A\sin B = \frac{1}{2}\left(\cos(A-B) - \cos(A+B)\right)$
37. $\cos A\cos B = \frac{1}{2}\left(\cos(A-B) + \cos(A+B)\right)$
38. $\sin A\cos B = \frac{1}{2}\left(\sin(A+B) + \sin(A-B)\right)$
39. $\cos A\sin B = \frac{1}{2}\left(\sin(A+B) - \sin(A-B)\right)$

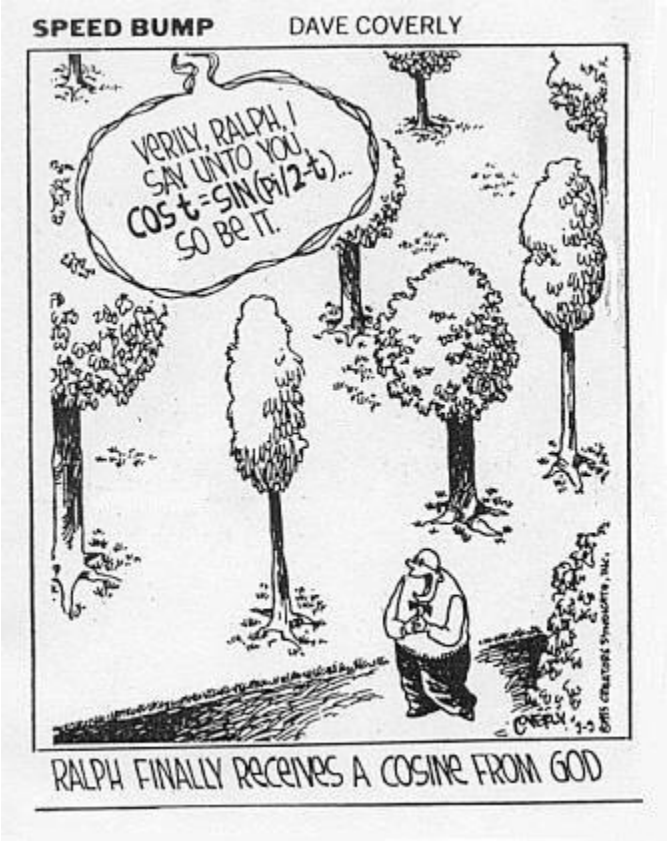
X. Sum-to-Product Identities

40. $\sin A + \sin B = 2\sin\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)$

41. $\sin A - \sin B = 2\cos\left(\frac{A+B}{2}\right)\sin\left(\frac{A-B}{2}\right)$

42. $\cos A + \cos B = 2\cos\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)$

43. $\cos A - \cos B = -2\sin\left(\frac{A+B}{2}\right)\sin\left(\frac{A-B}{2}\right)$



Some Other Trig Formulas

XI. Law of Sines

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

XII. Law of Cosines

$$c^2 = a^2 + b^2 - 2ab\cos C \qquad \cos C = \frac{a^2 + b^2 - c^2}{2ab}$$

$$a^2 = b^2 + c^2 - 2bc\cos A \qquad \cos A = \frac{b^2 + c^2 - a^2}{2bc}$$

$$b^2 = a^2 + c^2 - 2ac\cos B \qquad \cos B = \frac{a^2 + c^2 - b^2}{2ac}$$

XIII. Area of a Triangle

$A = \frac{1}{2}bh$ $\quad b$ = base, $\quad h$ = altitude

$A = \frac{1}{2}bc\sin A$ $\quad b, c$ are sides, A is the included angle

$A = \frac{1}{2}ab\sin C$ $\quad a, b$ are sides, C is the included angle

$A = \frac{1}{2}ac\sin B$ $\quad a, c$ are sides, B is the included angle

$A = \sqrt{s(s-a)(s-b)(s-c)}$ $\quad a, b, c$ are sides and s = semiperimeter

$$s = \frac{a+b+c}{2}$$