Using the Law of Sines and the Law of Cosines to Solve Triangles

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1. If *SSS*

   Given sides **a**, **b**, and **c**,
   Use the *Law of Cosines* to determine m$\angle$ **A**.

   1. Use the *Law of Cosines* to determine m$\angle$ **B**.
   2. Use the *sum of the angles of a triangle = 180°* to find m$\angle$ **C**.

2. If *SAS*
   1. Given sides **a** and **b**, and $\angle$ **C**,
      Use the *Law of Cosines* to determine side **c**.
   2. Use the *Law of Cosines* to determine $\angle$ **B**.
   3. Use the *sum of the angles of a triangle = 180°* to find m$\angle$ **A**.

3. If *ASA*
   1. Given m$\angle$ **A** and m$\angle$ **B** and side **c**,
      Use the *sum of the angles of a triangle = 180°* to find m$\angle$ **C**.
   2. Use the *Law of Sines* to determine side **b**.
   3. Use the *Law of Sines* to determine side **a**.

4. If *AAS*
   1. Given m$\angle$ **A** and m$\angle$ **B** and side **a**,
      Use the *sum of the angles of a triangle = 180°* to find m$\angle$ **C**.
   2. Use the *Law of Sines* to determine side **b**.
   3. Use the *Law of Sines* to determine side **c**.

5. If *SSA* (Ambiguous Case)
   1. Given sides **a** and **b**, and $\angle$ **A**,
      Use the *Law of Sines* to solve for sin $\angle$ **B**.
      1. If sin$\angle$ **B** > 1,
         There is **no** triangle.
      2. If sin$\angle$ **B** $\leq$ 1,
         Determine m$\angle$ **B** in quadrant I.

1. If m$\angle$ **A** + m$\angle$ **B** $\geq$ 180°
   There is **no** triangle.
2. If m$\angle$ **A** + m$\angle$ **B** < 180°
   There is at least one triangle.
   1. Determine m$\angle$ **B** in quadrant II.
      It has the same sine value as $\angle$ **B** .
      Call this angle, $\angle$ **B'**.
   2. Determine m$\angle$ **A** + m$\angle$ **B'**
      1. If m$\angle$ **A** + m$\angle$ **B'** $\geq$ 180°
         There is only one triangle.
         1. Determine m$\angle$ **C** using the *sum of the angles in a triangle = 180°*
         2. Determine side **c** using the *Law of Sines*.
      2. If m$\angle$ **A** + m$\angle$ **B'** < 180°
         There are two triangles.
         1. Determine m$\angle$ **C** using the *sum of the angles in a triangle = 180°*
         2. Determine side **c** using the *Law of Sines*.
         3. Determine m$\angle$ **C'** using the *sum of the angles in a triangle = 180°*
         4. Determine side **c'** using the *Law of Sines*.