Precalculus Final Examination Name _____

I. Definitions

1. Write the definition for a ***function***.

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2. Write the general form of a ***sinusoidal function***.

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3. Write the definition of the ***cosine*** function.

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_____

4. Write the general equation for an ***exponential function***.

_____

5. Write the definition of the ***mode*** of a set of data.

_____

6. What is the definition of the ***period*** of a trigonometric function?

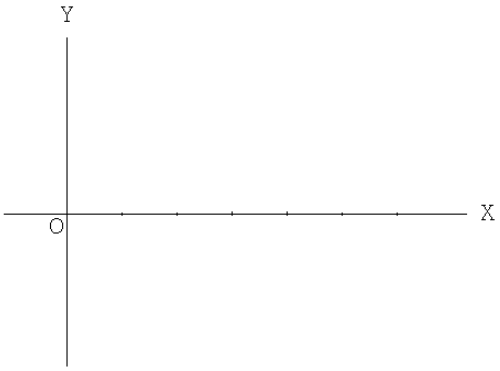
_____

II. Sketch the graphs of each of the following in the spaces provided.

7. A normal distribution.

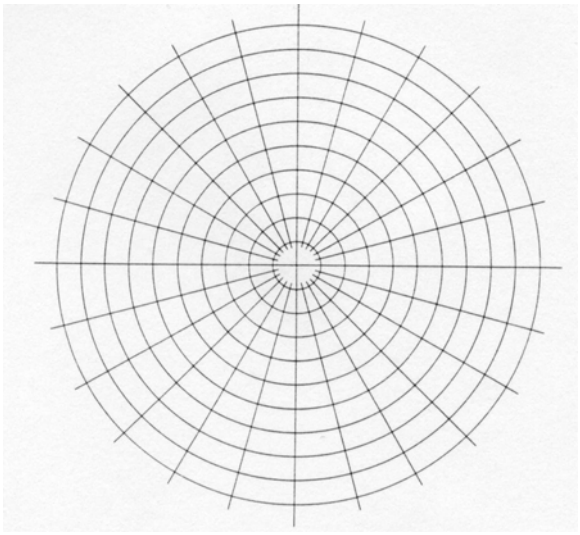
_____

8. The cotangent function.

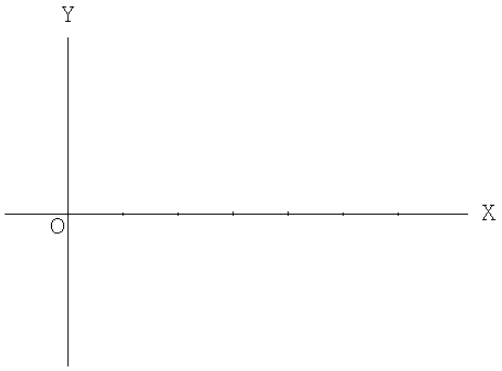


9. Graph the following points on the polar coordinate graph paper below:

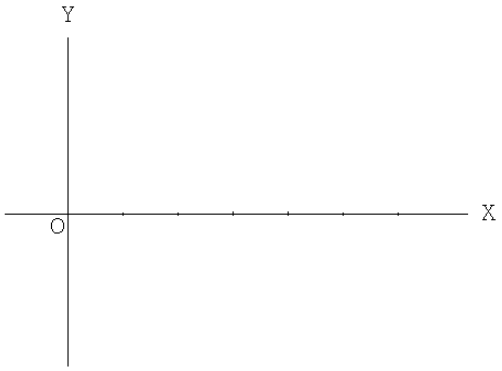
A (7, 45˚)
B (-5, -300˚)
C (4, -30˚)



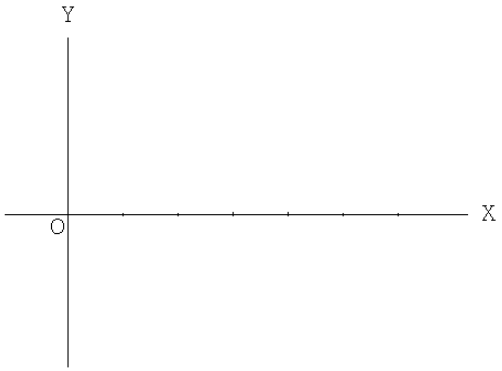
10. The secant function.



11. $y = 3\cos(2x)$



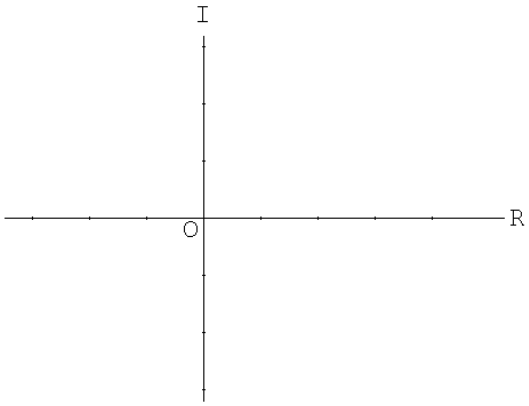
12. $y = Sin^{-1}x$



13. A rose curve with three petals.

14. Graph the following on the complex number plane:

(A) –2 + 4i (B) 3 + 2i (C) 1 – 3i



III. Different kinds of functions have different properties.

15. State the trigonometric identity involving the squares of secants and tangents.

_____

16. State the double angle property for **sin (2x)**.

_____

17. State the property of the logarithm of the product of two numbers.

_____

18. State the ***Law of Sines***.

_____

19. State the probability that event A occurs *or* that event B occurs.

_____

20. State ***Heron's formula***.

_____

IV. Proofs are essential in mathematics.

21. Prove by ***mathematical induction*** that the sum of the first n positive odd integers is n^2.

Prove: $1 + 3 + 5 + \ldots + (2n - 1) = n^2$

22. Prove the following identity:

$$\cos^4 \theta - \sin^4 \theta = 1 - 2\sin^2 \theta$$

23. Prove the following identity:

$$\frac{\tan^2 x + 6\tan x + 5}{\sec^2 x - 2} = \frac{\tan x + 5}{\tan x - 1}$$

23. Prove the following identity:

$$(1 + \sin\theta)(1 - \sin\theta) = \cos^2 \theta$$

V. The word *inverse* is an important concept in mathematics.

_____ 24. Determine the additive inverse of 7.

_____ 25. Determine the inverse function of $y = 4x - 12$

_____ 26. Determine the inverse tangent of –1
(that is, evaluate $\text{Tan}^{-1}(-1)$)

VI. Multiple Choice

_____ 27. A geometric series has $t_1 = 7$ and $r = 3$. Determine the value of the fifteenth term.

(A) 472,969 (B) 14,348,907
(C) 33,480,783 (D) 1.0044×10^8

_____ 28. The number 818 is a term in the arithmetic sequence
19, 36, 53, … Which term is it?

(A) 53 (B) 48 (C) 47 (D) 43

_____ 29. The binomial $(h - j)^{20}$ is expanded as a binomial series.
The term with j^7 is

(A) $-77520\ h^{13}\ j^7$ (B) $125970\ h^{13}\ j^7$
(C) $77520\ h^{13}\ j^7$ (D) $-125970\ h^{13}\ j^7$

_____ 30. If $\vec{a} = 8\vec{i} - 5\vec{j}$ and $\vec{b} = 6\vec{i} + 7\vec{j}$, determine $\vec{a} + \vec{b}$ in terms of its components.

(A) $14\vec{i} + 2\vec{j}$ (B) $14\vec{i} - 2\vec{j}$ (C) $48\vec{i} - 35\vec{j}$
(D) $-2\vec{i} + 12\vec{j}$ (E) None of these

_____ 31. Rewrite $7 - 2i$ in trigonometric form (Polar form).

(A) $\sqrt{53}(\cos 344.1^\circ + i \sin 344.1^\circ)$ (B) $3\sqrt{5}(\cos 344.1^\circ + i \sin 344.1^\circ)$
(C) $3\sqrt{5}(\cos 15.9^\circ + i \sin 15.9^\circ)$ (D) $\sqrt{53}(\cos 15.9^\circ + i \sin 15.9^\circ)$
(E) None of these

_____ 32. A function having the **period π** is

(A) $y = \sin(2\theta)$ (B) $y = \frac{1}{2}\sin(\theta)$

(C) $y = \sin\left(\frac{1}{2}\theta\right)$ (D) $y = 2\sin(\theta)$

_____ 33. Simplify: $\dfrac{1-\sin^2\theta}{\sin\theta}\cdot\dfrac{1}{\cos^2\theta}$

(A) $\csc(\theta)$ (B) $\sin(\theta)$ (C) $\cot(\theta)$ (D) $\cos^2\theta$

_____ 34. If $\cos\theta=-\dfrac{3}{5}$ and $\tan\theta=-\dfrac{4}{3}$ then $\sin\theta=$

(A) $\dfrac{3}{4}$ (B) $\dfrac{4}{5}$ (C) $-\dfrac{4}{5}$ (D) $-\dfrac{3}{4}$

VII. Short Answer

_____ 35. Solve for x: $x=Sin^{-1}\left(\dfrac{\sqrt{2}}{2}\right)$

_____ 36. Solve for θ: $2\cos^2\theta-5\cos\theta+2=0$ for $0^\circ\le\theta<360^\circ$

_____ 37. Solve for θ: $2\cos^2\theta-1=0$ for $0^\circ\le\theta<360^\circ$

_____ 38. Convert $\dfrac{8\pi}{15}$ radians to degrees.

_____ 39. Determine the value of $\sec(-2255^\circ)$

_____ 40. Which trigonometric functions are positive in the second quadrant?

_____ 41. In which quadrants is the cotangent negative?

_____ 42. A window is 23 feet above the ground. What angle will a 27-foot ladder make with the house when it is touched to the bottom of the window?

_____ 43. Determine the measure of the smallest angle in a 3-4-5 right triangle.

VIII. Multiple Choice

_____ 44. If θ is an acute angle, express $\sin\theta$ in terms of $\cos\theta$.

(A) $\sqrt{1-\cos^2\theta}$ (B) $\sqrt{\cos^2\theta-1}$
(C) $1-\cos\theta$ (D) $1-\cos^2\theta$

_____ 45. If sin(B) = cos(B), what is the measure of $\angle B$?

(A) $\frac{\pi}{6}$ (B) $\frac{\pi}{2}$ (C) $\frac{\pi}{3}$ (D) $\frac{\pi}{4}$

_____ 46. In $\triangle ABC$, side a = 3 inches, $\sin A = .1$, and $\sin B = .2$; what is the length of side b ?

(A) 6 (B) .6 (C) 1.5 (D) 15

_____ 47. The expression $\log\sqrt{xy}$ is equivalent to

(A) $2(\log x + \log y)$ (B) $\frac{1}{2}(\log x \cdot \log y)$
(C) $\frac{1}{2}(\log x + \log y)$ (D) $2\log x \cdot \log y$

_____ 48. The inverse of $y = \log_2 x$ is

(A) $y = x^2$ (B) $y = 2^x$ (C) $x = 2^y$ (D) $x = y^2$

_____ 49. In $\triangle ABC$, if side a = 8, side b = 9, and side c = 10, what is the measure of $\angle C$?

(A) 36.8° (B) 65.4° (C) 71.8° (D) 89.4°

_____ 50. Given $\triangle ABC$ with side a = 91.6 inches, side c = 24.19 inches, and $m\angle B = 37^\circ$, Determine the area of the triangle.

A) 1107.9 square inches B) 1333.5 square inches
C) 1769.6 square inches D) 666.8 square inches
E) None of these

_____ 51. In a baseball park, the distance from home plate to a point A in straightaway centerfield is 400 feet. Determine the distance from A to first base. (Straightaway centerfield is an extension of the line drawn from home plate through second base. The distance between consecutive bases is 90 feet).

(A) 378 feet (B) 66.2 feet (C) 135 feet (D) 342 feet

_____ 52. If $\log_x\left(\frac{1}{4}\right) = 2$ then x =

(A) 2 (B) -2 (C) $\frac{1}{2}$ (D) $-\frac{1}{2}$

_____ 53. Determine the value of i^{352}

(A) -1 (B) -i (C) i (D) 1

_____ 54. Determine the resultant of the two given displacements below:

8 units at a bearing of 90° followed by 6 units along a bearing of 210°

(A) 7.2 units at a bearing of 136°
(B) 51.9 units at a bearing of 46°
(C) 7.2 units at a bearing of 46°
(D) Can not be determined
(E) None of the above

_____ 55. Simplify $\frac{2}{3-i}$

(A) $\frac{3+i}{2}$ (B) $\frac{6-2i}{3-i}$ (C) $\frac{3+i}{5}$ (D) $\frac{3+i}{4}$

_____ 56. In ΔPEG, p = 6 cm, e = 7 cm, and g = 11 cm. Then $m\angle G =$
A) 115.3° B) 98.6° C) 64.7° D) 18.27°
E) Not possible (no such triangle)

_____ 57. Compute the number of <u>combinations</u> of 8 items taken 3 at a time.

(A) 336 (B) 56 (C) 40,320 (D) 6

IX. Problems (Short Answer)

_____ 58. Compute ${}_5P_3$

_____ 59. If a card is drawn at random from a standard deck of cards, calculate the probability of drawing an ace.

_____ 60. In how many ways could you arrange 4 books on a shelf if there are 10 books from which to choose?

_____ 61. There are 8 boys and 4 girls on the Handley golf team. The coach selects a group of 5 at random. What is the probability that the group consists of 3 boys and 3 girls?

62 – 64. Handley's Softball team, Boys' Track team, and Girls' Tennis team all play on Saturday. The probability that the track team will win is 0.6; the probability that the softball team will win is 0.5; and the probability that the tennis team will win is 0.8.

_____ 62. What is the probability that all three will win?

_____ 63. What is the probability that all three will lose?

_____ 64. What is the probability that all at least one of them will win?

_____ 65. On the mall at Apple Blossom, you were stunned to find your Precalculus teacher operating a table with the following game:

You pay $1.00 and then pick a card at random from a standard deck of 52 cards.
If the card is an ace, you win $4.00.
If the card is a face card, you win $2.00.
If the card is anything else, you win nothing.

What is your mathematical expectation?

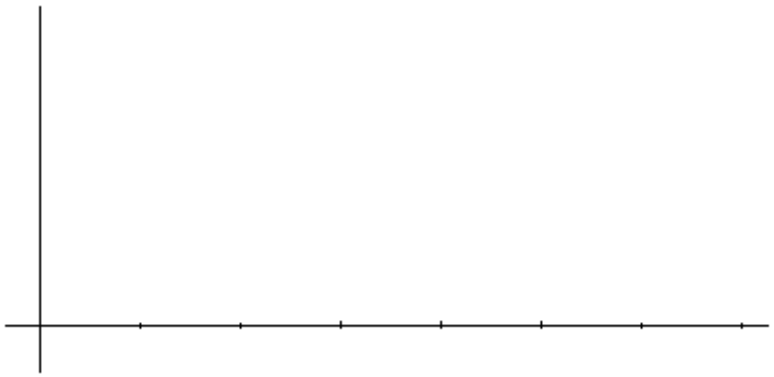
_____ 66. Mr. P has 5 sport jackets, 5 pairs of trousers, and 320 hideous ties. How many different outfits (1 tie, 1 jacket, and 1 pair of pants) could he choose from if he selects a jacket, a tie, and a pair of pants at random?

_____ 67. Winchester's favorite *Elvis* restaurant, *Red, Hot , and Blue*, offers three types of ribs (*Sweet Ribs, Dry Ribs, and Wet Ribs*) and four Memphis-style sandwiches (*Pulled Pork, Pulled Chicken, Beef Brisket, and Ribwich Combo*). In how many different ways can you select one of the Ribs meals **OR** one of the sandwiches?

68 –73. Given the following data:

x	Frequency
3	9
4	8
5	6
6	3
7	4

(histogram) 68. Plot a histogram of the frequency distribution.



_____ 69. Determine the **mean** for the set of data.

_____ 70. Determine the **median** for the set of data.

_____ 71. Determine the **mode** for the set of data.

_____ 72. Determine the **variance** for the set of data.

_____ 73. Determine the **standard deviation** for the set of data.

74 – 77. In 2001, 1,276,320 students took the SAT. The mean for the math SAT was 514 with a standard deviation of 113. Suppose that Matt E. Matics made a score of 640.

_____ 74. How many standard deviations is Matt's score above the mean?

_____ 75. What percent of the students made below Matt's score of 640?

_____ 76. How many of the 1,276,320 students scored higher than Matt?

_____ 77. What percent of the students scored between 490 and 700 ?

_____ 78. Transform the following to Cartesian coordinates and simplify.

$$r = 2\sin\theta$$

_____ 79. Transform the following to polar coordinates:

$$y = x^2$$

_____ 80. Transform the parametric equations to a Cartesian equation by eliminating the parameter.

$$x = 2t - 1$$
$$y = 3t + 1$$