

There is a formula for determining the number of combinations of n items if you take r of them at a time.

$${}_nC_r = \frac{n!}{r!(n-r)!}$$

If you are not familiar with the concept, let's take an example.

If there are 4 items called A, B, C, and D, and you want the number of combinations of any three of them, here are the possibilities:

ABC ABD ACD BCD.

So the answer is that there are 4 combinations of 4 items taken 3 at a time.

Here is the formula:

$${}_4C_3 = \frac{4!}{3!(4-3)!} = \frac{4 \times 3 \times 2 \times 1}{3 \times 2 \times 1 (1)} = 4$$

So, to find the number of possible combinations that Sonic could have for their 12 basic drinks and add-ins of up to 6 (out of 16 possibilities):

You must find the number of possible ways for each of the following:

No add-ins

1 add in

2 add ins

3 add ins

4 add ins

5 add ins

6 add ins

Then add them up.

No add-ins	-- there are 12 basic drinks:	12
1 add in	-- use ${}_{16}C_1$ to get 16, then multiply by 12 basic drinks to get	192
2 add ins	-- use ${}_{16}C_2$ to get 120, then multiply by 12 basic drinks to get	1440
3 add ins	-- use ${}_{16}C_3$ to get 560, then multiply by 12 basic drinks to get	6720
4 add ins	-- use ${}_{16}C_4$ to get 1820, then multiply by 12 basic drinks to get	21840
5 add ins	-- use ${}_{16}C_5$ to get 4368, then multiply by 12 basic drinks to get	52416
6 add ins	-- use ${}_{16}C_6$ to get 8008, then multiply by 12 basic drinks to get	96096

Total number of combinations		178716