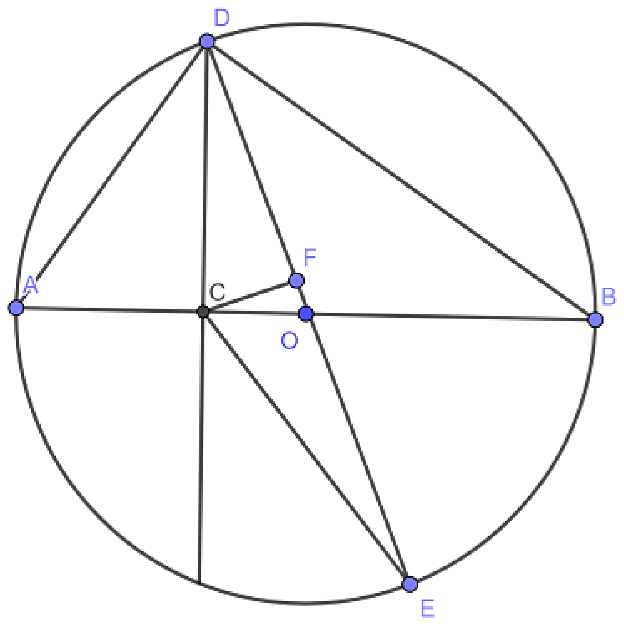
Circle Problem

Let r be the radius of the circle. Then. according to the problem statement, we can define (see the figure):



$$\overline{AB} = 2r$$

$$\overline{AC} = \frac{2r}{3}$$

$$\overline{OD} = r$$

$$\overline{OC} = r - \frac{2r}{3} = \frac{r}{3}$$

$$\overline{OD}^2 = \overline{CD}^2 + \overline{OC}^2$$

Substituting and resolving,

$$\overline{CD} = \frac{2\sqrt{2}r}{3}$$

Now, the area of ΔADB could be calculated as a function of r.

$$\frac{1}{2}(2r)\frac{2\sqrt{2}r}{3} = \frac{2\sqrt{2}r^2}{3}$$

By other hand, triangles ΔDCF and ΔCGD are similar (two right angles and one common angle), so, $\frac{\overline{CF}}{\overline{CG}} = \frac{\overline{CD}}{\overline{DG}}$. Substituting and resolving,

$$\overline{CF} = \frac{\overline{CG} \cdot \overline{CD}}{\overline{DG}}$$

$$\overline{CF} = \frac{(\frac{r}{3})(\frac{2\sqrt{2}r}{3})}{r} = \frac{2\sqrt{2}r}{9}$$

The area of $\triangle DCE$ could be calculated as

$$\frac{1}{2}\overline{DE}\cdot\overline{CF} = \frac{1}{2}(2r)\left(\frac{2\sqrt{2}r}{9}\right) = \frac{2\sqrt{2}r^2}{9}$$

Finally, the ratio of the area of $\triangle DCE$ to the area of $\triangle ADB$ is

$$\frac{\frac{2\sqrt{2}r^2}{9}}{\frac{2\sqrt{2}r^2}{3}} = \frac{1}{3}$$

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