

February 2024 Problem of the Month
Lucky Numbers

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Assume that C is telling the truth

	Lucky #	Unlucky #
A (liar)		
B (?)	6	
C (truth)	Not 7	
D (?)		
Missing # : not 5		

Assume that D is telling the truth

- If the product of B's numbers is 24, then B's Unlucky # must be 4.
 - This implies that sum of the lucky and unlucky # is 10
- A's lucky # is larger than any of the other numbers
 - This implies that A's lucky # is either 8 or 9

	Lucky #	Unlucky #
A (liar)	8 or 9	2 or 1
B (liar)	6	4
C (truth)	Not 7	Not 4
D (truth)	Not 2	
Missing # : not 5		

Because we must use 5 (from A's claim that the missing number is 5), and the sum must add up to 10, 5 can not belong to D since that would require the usage of 5 twice. Similarly, we also can not use it for C. Hence there is no spot left for 5 to go which means D is a liar.

Hence, we know that if C is telling the truth then A and D are liars while B and C are truth teller.

Filling out from given information so far:

	Lucky #	Unlucky #
A (liar)	Not largest (not 8,9)	
B (truth)	6	(not 4)
C (truth)	(not 7)	4
D (liar)	2	
Missing # : not 5		

Recall :

- The sum of lucky and unlucky numbers is the same for all four persons
 - D's unlucky number can not be 1,3 since C's sum is at least 5.
 - Sum of lucky number and unlucky number can not be 10 or 11
 - D's unlucky number can not be 8 or 9

	Lucky #	Unlucky #
A (liar)	Not largest (not 8,9)	
B (truth)	6	(not 4)
C (truth)	(not 7)	4
D (liar)	2	(not 1,3,8,9)
Missing # : not 5		

Assuming that D's unlucky number is 5 :

- A must use 7,8,9 to total to 7, which is impossible

	Lucky #	Unlucky #
A (liar)	7	(8, or 9)
B (truth)	6	1
C (truth)	3	4
D (liar)	2	5
Missing # : not 5		

Assuming that D's unlucky number is 7 :

- 2,3,4,5,6,7 gets used ; so A must use 1,8,9 to total to 9. Since A's lucky number can not be 8 or 9, it must be 1; hence unlucky number is 8, and missing number is 9

	Lucky #	Unlucky #
A (liar)	1	8
B (truth)	6	3
C (truth)	5	4
D (liar)	2	7
Missing # : 9		

The sum of the lucky numbers is less than the sum of the unlucky numbers, so C must be a liar

Assume that A is telling the truth

	Lucky #	Unlucky #
A (truth)		
B (?)	Not 6	
C (liar)	7	
D (?)		
Missing # : 5		

Assume that B is telling the truth :

In the end, you end up having to make 2 pairs of 11 while only using 1,3,6, or 8 once which is not possible, so B must be a liar

	Lucky #	Unlucky #
A (truth)	1,3,6	1,3,6,8
B (truth)	1,3,8	1,3,6
C (liar)	7	4
D (liar)	2	9
Missing # : 5		

By process of elimination of liars and truth we found:

	Lucky #	Unlucky #
A (truth)	Largest, 8 or 9	
B (liar)	Not 6	Multiply to 24
C (liar)	7	Not 4
D (truth)	Not 2	
Missing # : 5		

Note that

- Only possible pairs of B is (4,6)
- Missing number is 5, and A's lucky number is larger than any of the other numbers, so it must be 9
- Sums of the lucky and unlucky numbers are the same for all four persons.
 - C's unlucky number is 3
 - A's unlucky number must be 1

	Lucky #	Unlucky #
A (truth)	9	1
B (liar)	4	6
C (liar)	7	3
D (truth)	Not 2	
Missing # : 5		

The only missing pair is (2,8) or (8,2), and lucky number can not be 2 so it is (8,2)

	Lucky #	Unlucky #
A (truth)	9	1
B (liar)	4	6
C (liar)	7	3
D (truth)	8	2
Missing # : 5		

Answer :

Lucky and unlucky numbers for each of them is the following :

	Lucky #	Unlucky #
A (truth)	9	1
B (liar)	4	6
C (liar)	7	3
D (truth)	8	2
Missing # : 5		