

Coloring Maps

Mapmakers follow two rules in coloring maps:

- Each country or state should be colored with a single color.
- Different colors should be used for countries or states that share a common border.

Figure 1 is a map of Utah, Colorado, Arizona, and New Mexico. We could use a different color for each state, but following the mapmakers' rules, we can use fewer colors. Color the map with the fewest possible colors. If the states touch at only a single point, they can be of the same color.

1. How many colors are required? _____



Fig. 1



Fig. 2

Figure 2 is a map of another portion of the western United States. How many colors do we need for this map if we follow the mapmakers' rules?

2. Are two colors enough? _____
3. If not, can the map be colored with three colors?

4. What is the minimum number of colors required to color the map that combines figures 1 and 2?

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Coloring Maps—Continued

Test your conjecture by coloring figure 3.



Fig. 3

For each of the regional maps in figure 4, determine the fewest colors necessary to color the states following the mapmakers' rules.

5. Which maps require only three colors? _____
6. Which maps require four colors? _____
7. Do any require five colors? _____
8. Is it possible for a map to require five colors? _____

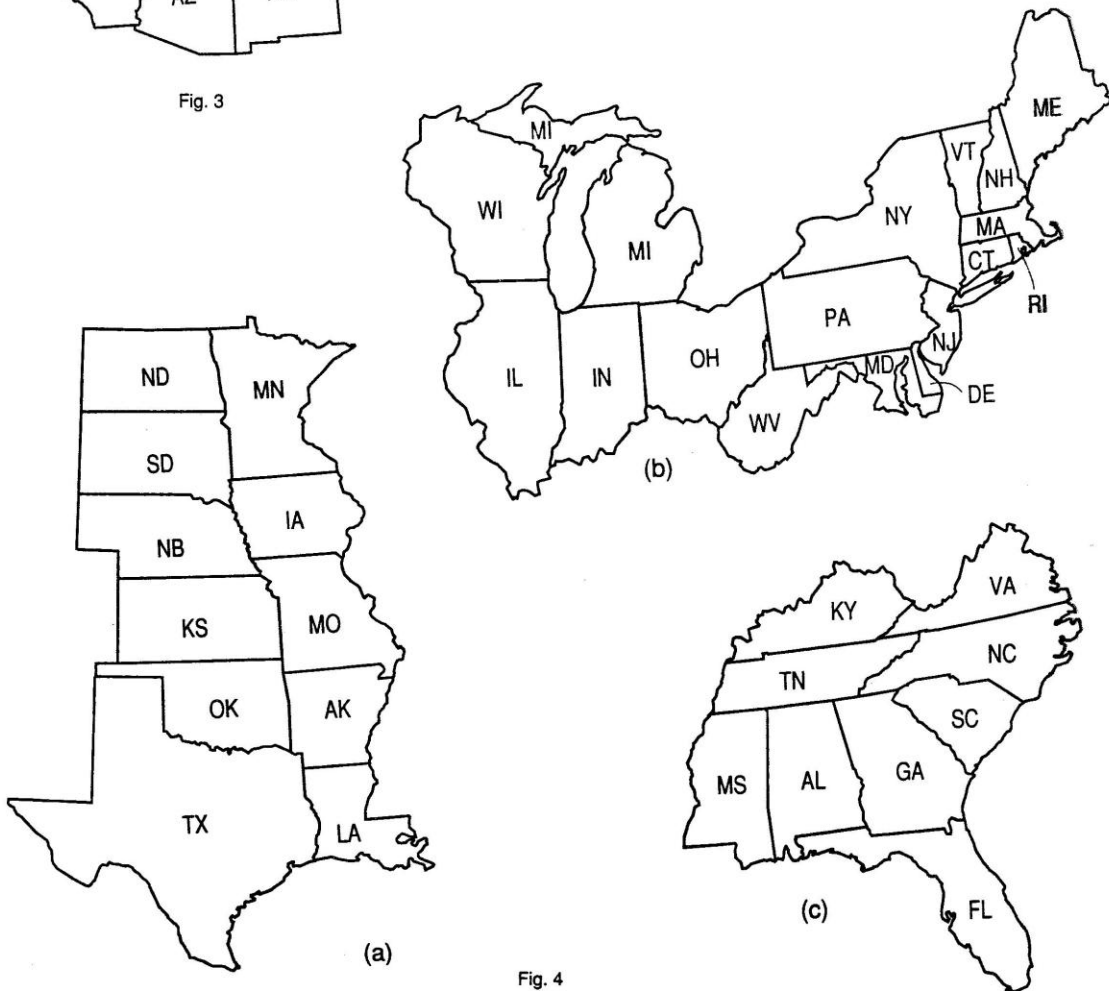


Fig. 4

Coloring Maps—Continued

9. Consider figure 5, in which country A is separated into two pieces. Are five colors necessary? _____
10. Are any states in the United States broken into more than one piece? _____ Which ones? _____
11. Is it possible to create a map that would require six colors? _____
12. Is it possible to create a map requiring five colors in which none of the countries are broken? _____

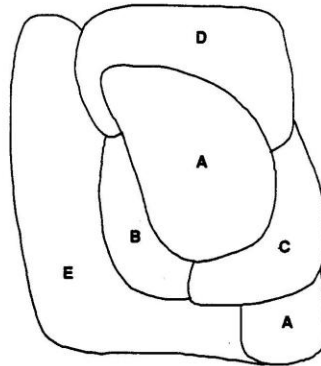


Fig. 5

Figure 6 shows a map of the continental United States. Four colors are enough to color this map. Color it using the mapmakers' rules.

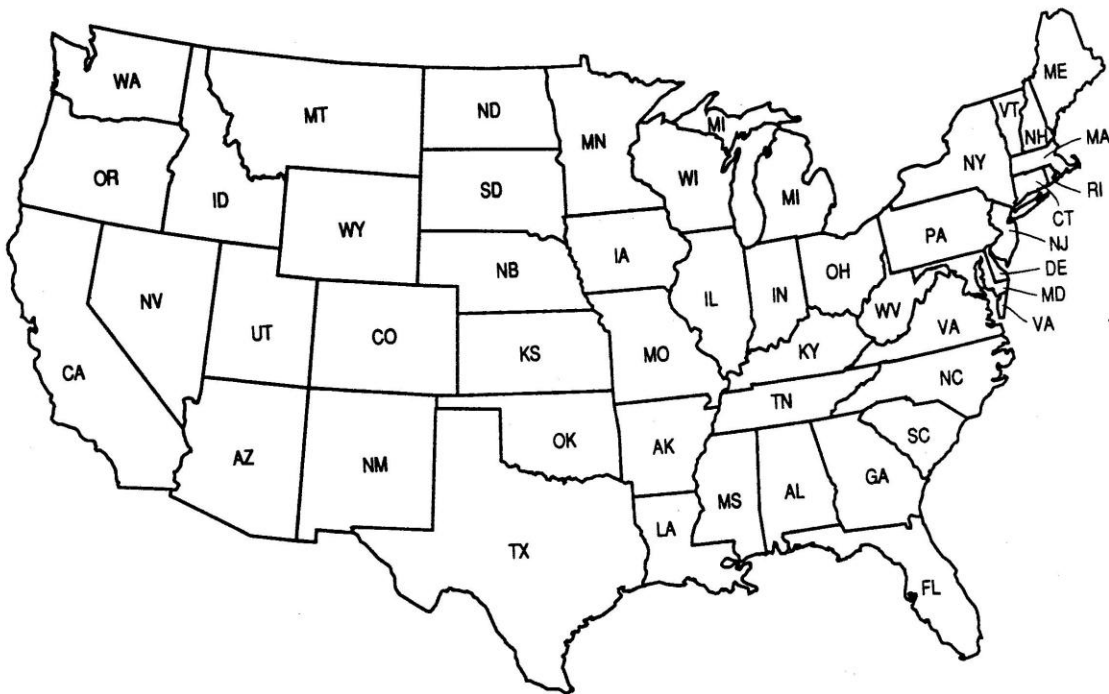
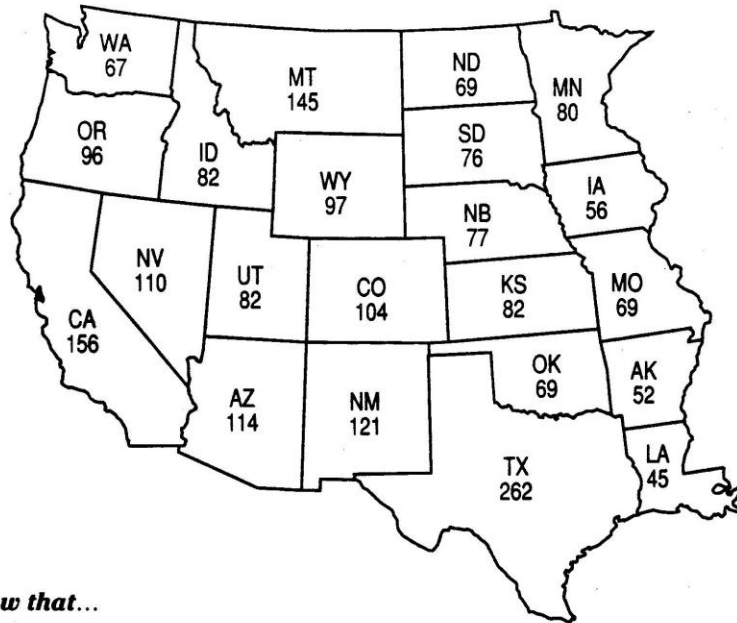


Fig. 6

Can you...

- color the map below at a minimum cost if the four chosen colors cost \$1, \$2, \$3, and \$4 per 1000 square miles, respectively? The number in each state represents thousands of square miles.



Did you know that...

- if a map is drawn on a flat plane or a globe and if all the countries or states are single, unbroken regions, then four colors always suffice to color the map (four-color theorem)?
- the four-color theorem was verified in 1976 at the University of Illinois with the help of computers?
- if we draw our map on a doughnut, the map may require as many as seven different colors?

Bibliography

- "Thinking with Ink." In Problem-solving Strategies. (Computer program) Available from Minnesota Educational Computing Corporation, 3490 Lexington Avenue North, St. Paul, MN 55126. Challenges students to color maps with inks of varying costs.

Answers:

- | | | | |
|--------|------------|----------|----------------------------------|
| 1. Two | 4. Four | 7. No | 10. Yes; Michigan and Virginia |
| 2. No | 5. a, b, c | 8. Maybe | 11. Yes |
| 3. Yes | 6. None | 9. Yes | 12. Not if the map is in a plane |

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