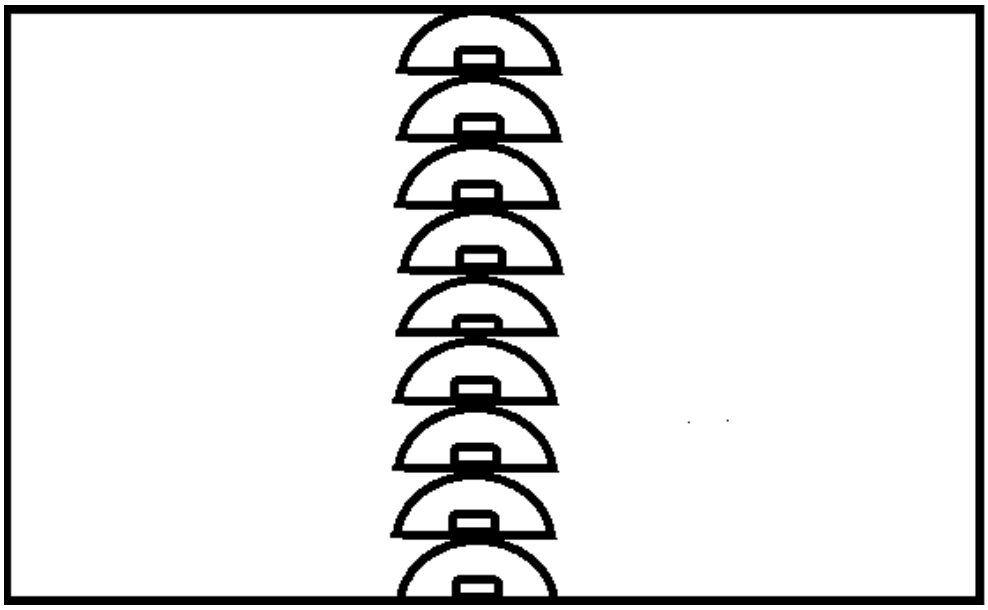
A Droodle for the S.A.T. Math Exam without a calculator

A puzzle by David Pleacher



"A Droodle is a borkley looking sort of drawing that doesn't make any sense until you know the correct title." – Roger Price

Caption for the picture:

E	S	K	I	M	O		A	P	A	R	T	M	E	N	T		H	O	U	S	E
20	13	12	1	9	18		16	8	7	17	4	11	14	5	2		19	3	15	6	10

I 1. On Sunday afternoon, Jackson sent m text messages each hour for 5 hours, and Kate sent p text messages each hour for 4 hours. Which of the following represents the total number of messages sent by Jackson and Kate on Sunday afternoon?

Jackson sent m messages for 5 hours for a total of $5m$ messages while Kate sent p messages for 4 hours for a total of $4p$ messages.

So the answer is $5m + 4p$ messages.

T 2. $g(x) = ax^2 + 24$

For the function $g(x)$, a is a constant and $g(4) = 8$.

What is the value of $g(-4)$?

Substitute 4 for x in the original function:

$g(4) = a(4)^2 + 24$ and set it equal to 8.

$16a + 24 = 8$. So a = -1.

Therefore, $g(-4) = (-1)(-4)^2 + 24 = -16 + 24 = 8$.

O 3. $\left(x^2y - 3y^2 + 5xy^2\right) - \left(-x^2y + 3xy^2 - 3y^2\right)$

Which of the following is equivalent to the preceding expression?

Collect li x^2y ke terms:

$x^2y + x^2y - 3y^2 + 3y^2 + 5xy^2 - 3xy^2$

$2x^2y + 2xy^2$

T 4. If $\frac{y-1}{3} = h$ and $h = 3$, what is the value of y?

Substitute 3 for h: $\frac{y-1}{3} = 3$

Then multiply by 3: $y - 1 = 9$, so $y = 10$

N 5. $x - 2y = 19$

$3x + 4y = -23$

What is the solution (x, y) of the system of equations?

Multiply the top equation and 3, then subtract the bottom equation:

$3x - 6y = 57$

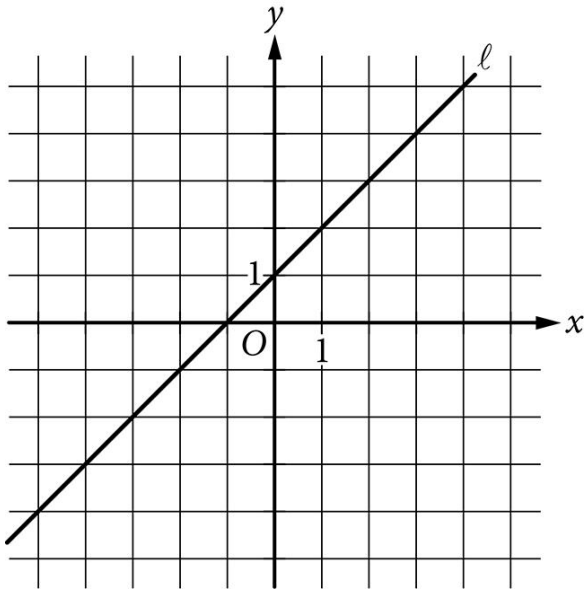
$3x + 4y = -23$

$-10y = 80$

$y = -8$

$x = 3$

S 6. Refer to the figure below:



Which of the following is an equation of the line ℓ ?

The slope is 1 and the y-intercept is 1, so the equation is
$y = x + 1$

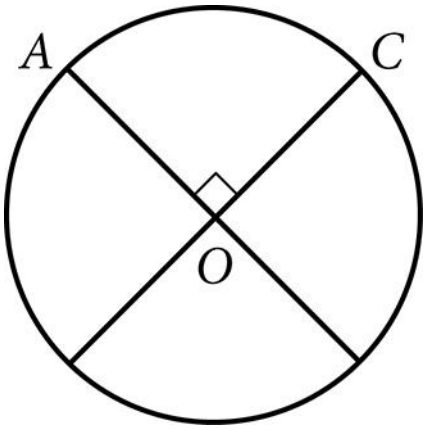
A 7. What are the solutions of the quadratic equation $4x^2 - 8x = 12$?

Divide each side by 4 to get $x^2 - 2x - 3 = 0$
Factor to get $(x - 3)(x + 1)$
So, $x = -1$ and $x = 3$

P 8. Which of the following is an example of a function whose graph has no x intercepts?

A quadratic function with no real zeros .
That means the function does not intersect the x-axis.

M 9. Refer to the figure below.



The circle above with center O has a circumference of 36. What is the length of minor arc AC ?

The arc AC represents ¼ of the circle, so the length of the arc is ¼ of the circumference = ¼ (36) = 9.

E 10. The volume of right circular cylinder A is 22 cubic centimeters. What is the volume, in cubic centimeters, of a right circular cylinder with twice the radius and half the height of cylinder A?

The volume of a cylinder is given by the formula: $V = \pi r^2 h$.

Now take twice the radius and half the height to get: $V_{\text{new}} = \pi (2r)^2 \frac{h}{2}$

Simplify to get $V_{\text{new}} = 2\pi r^2 h$

Since the new volume is twice as large as the original volume, the new volume is 2 x 22 = 44 cubic centimeters.

M 11. The expression $\dfrac{x^{-2}y^{\frac{1}{2}}}{x^{\frac{1}{3}}y^{-1}}$ where x >1 and y>1, is equivalent to which of the following?

$$\frac{x^{-2}y^{\frac{1}{2}}}{x^{\frac{1}{3}}y^{-1}} = \frac{y^{\frac{1}{2}}y}{x^2x^{\frac{1}{3}}} = \frac{y\sqrt{y}}{x^2\sqrt[3]{x}}$$

K 12. Which of the following expressions is equivalent to $\dfrac{x^2-2x-5}{x-3}$?

Divide using long division:

$$\begin{array}{r} x+1 \\ x-3\overline{)\,x^2-2x-5} \\ \underline{x^2-3x} \\ +x-5 \\ \underline{x-3} \\ -2 \end{array}$$

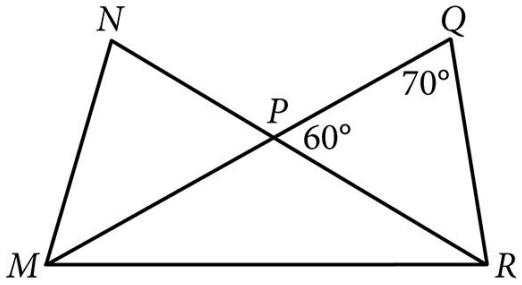
This is equivalent to $x+1-\dfrac{2}{x-3}$

S 13. The expression $\frac{1}{3}x^2-2$ can be rewritten as $\frac{1}{3}(x-k)(x+k)$, where *k* is a positive constant. What is the value of *k* ?

$$\frac{1}{3}x^2-2=\frac{1}{3}\left(x^2-6\right)=\frac{1}{3}\left(x-\sqrt{6}\right)\left(x+\sqrt{6}\right)$$

So, $k=\sqrt{6}$

E 14. In the figure below, $\overline{MQ}$ and $\overline{NR}$ intersect at point *P*, *NP* = *QP*, and *MP* = *PR*. What is the measure, in degrees, of $\angle QMR$?

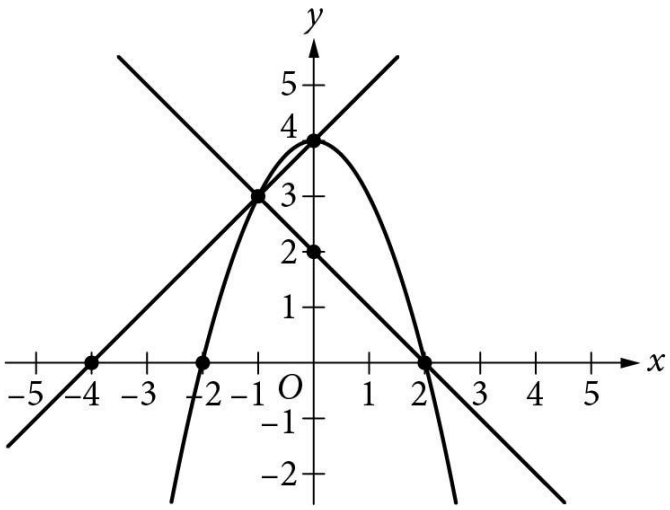


$m\angle MPR = 120°$ since it is a supplement to 60 degrees.
Since *MP* = *PR*, that makes $m\angle PMR = m\angle PRM$
So, $\angle QMR$ = 30 degrees.

U 15. The number of radians in a 720 degree angle can be written as $a\pi$, where *a* is a constant. What is the value of *a*?

$180° = \pi$ radians, so $720° = 4x180° = 4\pi$ radians

A 16.



A system of three equations is graphed above in the x y plane.
How many solutions does the system have?
Just one point where all three equations intersect (-1, 3)

R 17. Which of the following represents all the possible values of x that satisfy the following equation?

$$\frac{x}{x-3}=\frac{5x}{5}$$

Divide 5x by 5 to get *x*. The multiply each side by *x* – 3:

$x^2-3x=x$

$x^2-4x=0$

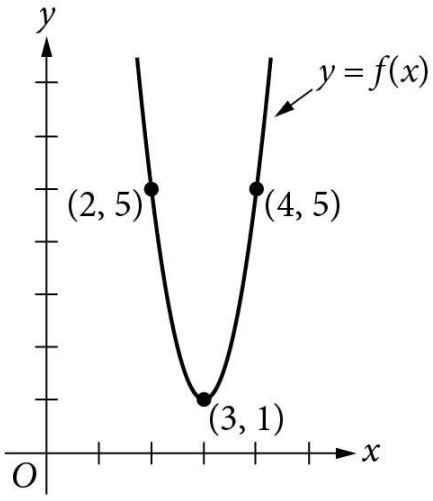
$x(x-4)=0$

So, x = 0 and 4

O 18. Given the expression $\frac{1}{2x+1}+5$ for x > 0, which of the following is an equivalent expression?

$$\frac{1}{2x+1}+5=\frac{1}{2x+1}+\frac{10x+5}{2x+1}=\frac{10x+6}{2x+1}$$

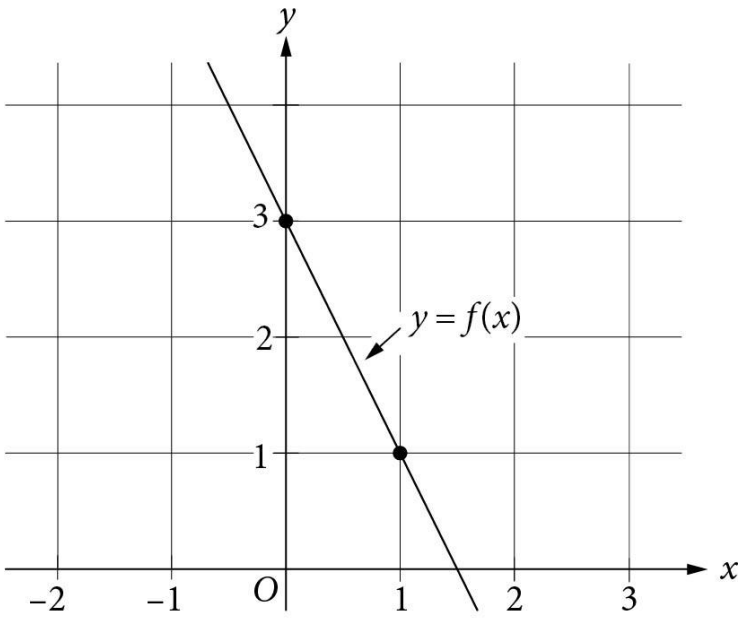
H 19. The graph of the function f below is a parabola. Which of the following defines f ?



Answer is $f(x)=4(x-3)^2+1$

E 20. The graph of the linear function f is shown in the following diagram. The graph of the linear function g (not shown) is perpendicular to the graph of f and passes through the point (1, 3).

What is the value of $g(0)$?



The slope of $f(x) = -2$, so the slope of $g(x) = \frac{1}{2}$.

Since it passes through the point $(1,3)$,

$$g(x) - 3 = \frac{1}{2}(x - 1)$$

$$g(x) = \frac{1}{2}x + 2\frac{1}{2}$$

$$g(0) = 2.5$$