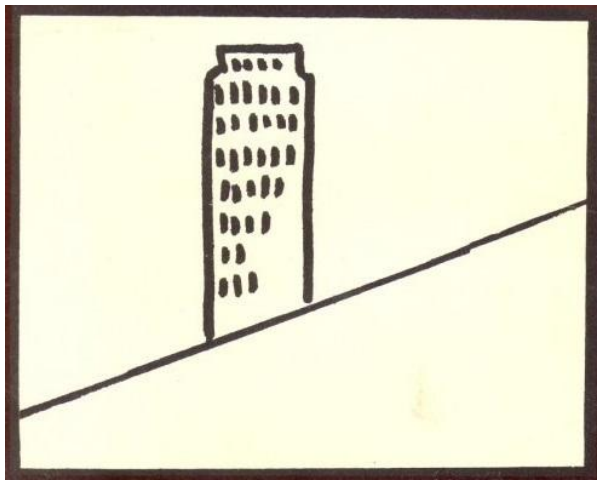
A Droodle for the S.A.T. Math Exam without a calculator

A puzzle by David Pleacher



"A Droodle is a borkley looking sort of drawing that doesn't make any sense until you know the correct title." – Roger Price

Caption for the picture:

T	O	W	E	R		O	F		P	I	S	A		A	S		S	E	E	N		B	Y
__	__	__	__	__		__	__		__	__	__	__		__	__		__	__	__	__		__	__
24	15	11	14	7		23	10		16	1	25	18		6	22		25	5	14	2		9	13

L	E	A	N	I	N	G		T	O	U	R	I	S	T
__	__	__	__	__	__	__		__	__	__	__	__	__	__
17	5	20	21	1	19	3		24	23	12	7	1	8	4

To determine the title to this droodle, solve the 25 math problems which are similar to problems found on the S.A.T. math section.
Then find the answers to each problem from the choices listed.
Replace the numbered blank with the letter corresponding to the answer for that problem.
A calculator should not be used on this part of the exam.

I 1. If $3r = 18$, what is the value of $6r + 3$?

If $3r = 18$, then $r = 6$.
So, $6r + 3 = 6\,(6) + 3 = 39$

_N_ 2. Which of the following is equal to $a^{\frac{2}{3}}$, for all values of *a* ?

N. $\sqrt[3]{a^2}$

_G_ 3. Given the system of equations $\begin{cases} 3x-2y=-6 \\ 2x-3y=-14 \end{cases}$.

If (x, y) is a solution of the system, what is the value of $x-y$?

The quickest way to solve is just to add the two equations, then divide each side by 5:

$5x-5y=-20$

$x-y=-4$

You could also solve the system for *x* and *y* and then subtract the two variables.

_T_ 4. The function *f* is defined by a polynomial. Some values of *x* and $f(x)$ are shown in the table below. Which of the following must be a factor of $f(x)$?

x	$f(x)$
0	3
2	1
4	0
5	−2

If $x-b$ is a factor of $f(x)$, then $f(b)$ must equal 0.

Based on the table, $f(4)$ equals 0. Therefore, $x-4$ must be a factor of $f(x)$.

_E_ 5. The parabola with equation $y=(x-11)^2$ intersects the line with equation $y=25$ at two points, *A* and *B*. What is the length of $\overline{AB}$?

$(x-11)^2=25$

$x-11=\pm 5$

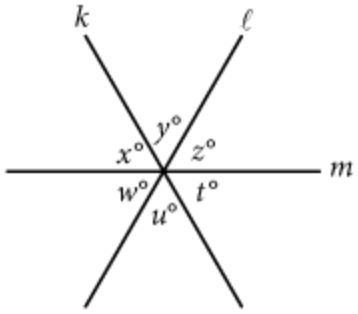
$x-11=-5$ and $x-11=5$

$x=6,16$

$AB=10$

A 6. In the diagram below, lines *k*, *l*, and *m* intersect at a point. If $x + y = u + w$, which of the following must be true?

I. x = z
II. y = w
III z = t



$x + y = u + w$

Vertical angles: y = u, x = t, w = z
Then z = t and x = w, so x = z

Answer is A. I and III only

R 7. The equation $\frac{24x^2 + 25x - 47}{ax - 2} = -8x - 3 - \frac{53}{ax - 2}$ is true for all values of $x \neq \frac{2}{a}$, where *a* is a constant. What is the value of *a* ?

In order for -8*x* - 3 to be a factor of the division problem, *a* must equal -3.
-8 times *a* must equal 24.

You could also divide using long division.

S 8. The equation $C = \frac{5}{9}(F - 32)$ shows how a temperature *F*, measured in degrees Fahrenheit, relates to a temperature *C*, measured in degrees Celsius. Based on the equation, which of the following must be true?

I. A temperature increase of 1 degree Fahrenheit is equivalent to a temperature increase of $\frac{5}{9}$ degree Celsius.

II. A temperature increase of 1 degree Celsius is equivalent to a temperature increase of 1.8 degrees Fahrenheit.

S. I and II only

_B_ 9. What is the sum of the complex numbers $2+3i$ and $4+8i$?

B. 6 + 11*i*

_F_ 10. In the equation $4x^2-9=(ax+m)(ax-m)$, *a* and *m* are constants. Which of the following could be the value of *a* ?

$4x^2=(ax)^2=a^2x^2$

$4=a^2$

$a=-2$ or $a=+2$

Therefore, the answer is F. 2

_W_ 11. If $\sqrt{x}+\sqrt{9}=\sqrt{64}$, what is the value of *x* ?

The easiest way to solve this problem is to evaluate the two square roots to get:

$\sqrt{x}+\sqrt{9}=\sqrt{64}$

$\sqrt{x}+3=8$

$\sqrt{x}=5$

$x=25$

_U_ 12. In the equation $2x^2-4x=k$, *k* is a constant. If the equation has no real solutions, Which of the following could be the value of *k* ?

If the equation has no real solutions then b^2-4ac must be less than zero.

$2x^2-4x=k \quad \Rightarrow \quad 2x^2-4x-k=0$

$b^2-4ac=(-4)^2-4(2)(-k)$

$=16+8k$

$\therefore \quad 16+8k<0$

$8k<-16$

$k<-2$

The only choice which is less than -2 is U. -3

__Y__ 13. A laundry service is buying detergent and fabric softener from its supplier. The supplier will deliver no more than 300 pounds in a shipment.

Each container of detergent weighs 7.35 pounds, and each container of fabric softener weighs 6.2 pounds.

The service wants to buy at least twice as many containers of detergent as containers of fabric softener.

Let *d* represent the number of containers of detergent, and let *s* represent the number of containers of fabric softener, where d and s are nonnegative integers.

Which of the following systems of inequalities best represents this situation?

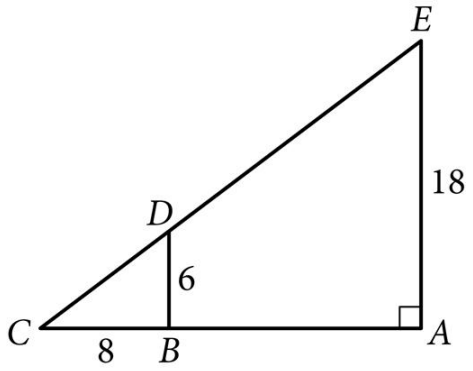
Y. $\begin{aligned} 7.35d + 6.2s &\le 300 \\ d &\ge 2s \end{aligned}$

__E__ 14. Which of the following is equivalent to $\left(a+\frac{b}{2}\right)^2$?

$$\left(a+\frac{b}{2}\right)^2 = a^2 + 2(a)\left(\frac{b}{2}\right) + \left(\frac{b}{2}\right)^2$$

$$= a^2 + +ab + \frac{b^2}{4}$$

__O__ 15. In the figure below, $\overline{BD}$ is parallel to $\overline{AE}$. What is the length of $\overline{CE}$?



$\triangle AEC$ and $\triangle BDC$ are similar right triangles and the ratios of their corresponding sides are equal. Also, you can use the Pythagorean theorem to solve for missing sides.

$CE = 10$ (Pythagorean theorem)

$\frac{6}{18} = \frac{10}{CE}$ (similar triangles)

$\therefore CE = 30$

_P_ 16. How many liters of a 25% saline solution must be added to 3 liters of a 10% saline solution to obtain a 15% saline solution?

Let x = number of liters of 25% saline solution

Then you can write the following equation:

$$.25x+(3)(.10)=(x+3)(.15)$$

$$.25x+.3=.45+.15x$$

$$.10x=.15$$

$$x=1.5$$

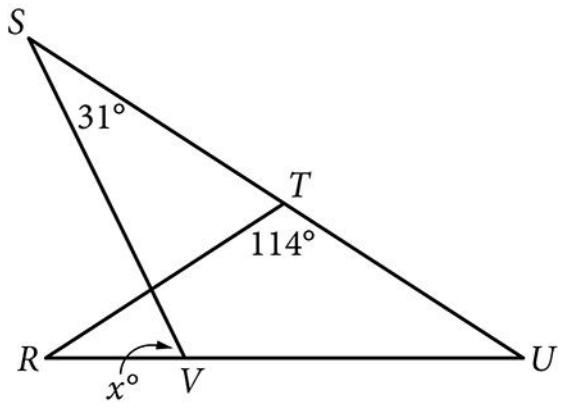
_L_ 17. Points *A* and *B* lie on a circle with radius 1, and arc AB has length $\frac{\pi}{3}$. What fraction of the circumference of the circle is the length of arc AB ?

$$C=2\pi r$$

$$C=2\pi(1)=2\pi$$

So, the length of arc AB is $\frac{\frac{\pi}{3}}{2\pi}=\frac{\pi}{3}x\frac{1}{2\pi}=\frac{1}{6}$ of the circumference.

_A_ 18. In the figure below, $RT=TU$. What is the value of x ?



Since $RT=TU$, then $\angle R\cong\angle U$.

Since the measures of the angles of a triangle add up to 180,

Then $m\angle R+m\angle U=66°$

Since the angles have equal measures, $m\angle R=m\angle U=33°$

$\angle RVS$ is an exterior angle of $\triangle SVU$

$m\angle RVS=m\angle S+m\angle U=31°+33°=64°$

_N_ 19. The width of a rectangular dance floor is w feet. The length of the floor is 6 feet longer than its width. Which of the following expresses the perimeter, in feet, of the dance floor in terms of w ?

Perimeter = $2l+2w$

$$P=2(w+6)+2w$$
$$=2w+12+2w=4w+12$$

_A_ 20. Which of the following expressions is equivalent to $\frac{f(x)}{g(x)}$ for $x>3$, where $f(x)=x^3-9x$ and $g(x)=x^2-2x-3$?

$$\frac{f(x)}{g(x)}=\frac{x^3-9x}{x^2-2x-3}=\frac{x(x^2-9)}{(x-3)(x+1)}=\frac{x(x-3)(x+3)}{(x-3)(x+1)}=\frac{x(x+3)}{(x+1)}$$

_N_ 21. The graph of $(x-6)^2+(y+5)^2=16$ is a circle. Point *P* is on the circle and has coordinates $(10,-5)$. If $\overline{PQ}$ is a diameter of the circle, what are the coordinates of point *Q*?

Since the center of the circle is $(6,-5)$ and point *P* has coordinates $(10,-5)$, and the raidus is 4, the point *Q* must have coordinates $(2,-5)$ -- four units on the other side.

_S_ 22. If $u+t=5$ and $u-t=2$, what is the value of $(u-t)(u^2-t^2)$?

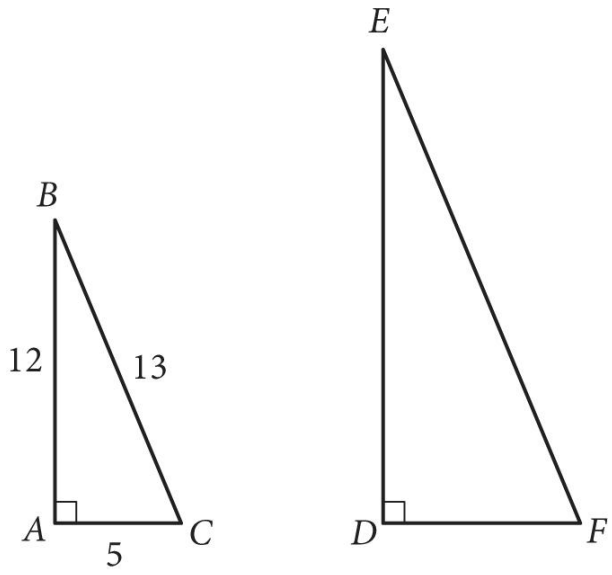
$$(u-t)(u^2-t^2)=(u-t)(u-t)(u+t)=(u-t)^2(u+t)$$
$$\therefore (u-t)^2(u+t)=(2)^2(5)=20$$

_O_ 23. In the system of equations below, *a* is a constant. For which of the following values of *a* does the system have no solution?

$$\begin{cases}-3x+y=6\\ ax+2y=4\end{cases}$$

To have no solution, the lines must be parallel and therefore, have the same slope. Since the slope of the first equation is *m = 3* then a must equal -6.

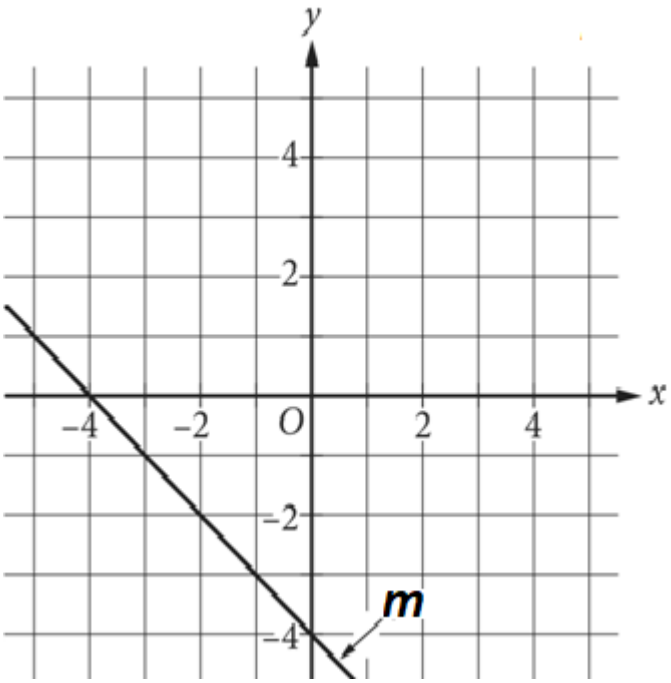
T 24. In the figure below $\triangle ABC$ is similar to $\triangle DEF$. What is the value of $\cos(E)$?



Since the triangles are similar, the sides are in the same ratio, so

$$\cos(E) = \cos(B) = \frac{12}{13}$$

_S_ 25. Which of the following is an equation of line ***m*** in the figure below ?



From the graph, you can see that the slope is -1 and the y-intercept is -4.
So the equation is $y = -1x - 4$.
Adding x to both sides, you obtain the answer: S. $x + y = -4$

The droodle used in this puzzle was drawn by Roger Price and appeared in his book called *Droodles.*