

# Tic-Tac-Toe in Polar Coordinates

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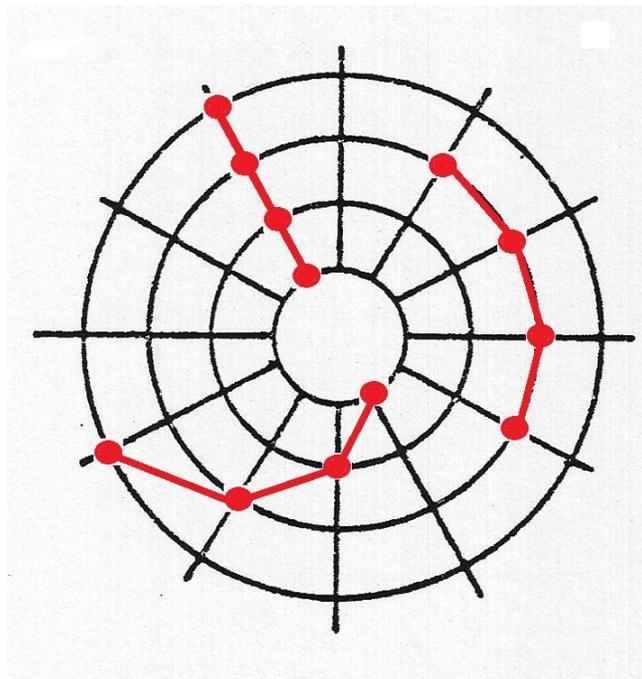
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The game of tic-tac-toe has long been used as a motivational device in mathematics instruction. Students can play by using numbers to create magic squares, they can study the strategy, they can create new strategies by playing to lose, or they can try to generalize the game to larger arrays or three dimensions. Kenneth Hummel invented a variation to help introduce point naming in polar coordinates. The game is described below using radian measures for angles. With younger students, degree measure is suitable too.

Polar Coordinate tic-tac-toe is played on the polar-coordinate system using circles of radius 1, 2, 3, and 4 and radial lines every  $\pi/6$  radians or 30 degrees. The origin is omitted as a playing point because its position is unfairly advantageous. Players alternate putting X and O at intersections of the lines and circles. The winner is the first player to place four of her marks in a row along (a) a radial line, (b) a circle, or (c) a diagonal or spiral (see figure below).



There are some simplified versions of the game. The size of the coordinate system may be reduced to three rings, with only three marks in a row required to win. Radial lines every  $\pi/4$  radians or 45 degrees are sufficient. Spirals may be deleted from the set of winning configurations.

The use of this game for introducing point naming in polar coordinates provides an excellent opportunity for the students to learn by discovery. As the students seem to catch on, change the rules slightly, requiring that the second component of the ordered pair be negative. Later, the same requirement may be placed on the first component or both components. As more students become competent at naming points, the class may be divided into smaller groups playing several separate games.

On the next page, you will find nine tic-tac-toe boards for polar coordinates.

