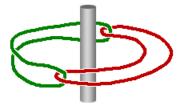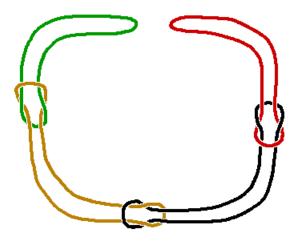# The Rubber-Band Problem
by Rod Fletcher
December 24, 2020

Given an unlimited supply of identical rubber bands, is it possible to entangle some of them so as to make a band (i.e., a single closed loop) longer than one of the original bands? Cutting them or treating them as one-dimensional strings and tying knots is prone to unraveling, so that's not allowed. Of course, any method of joining the rubber bands can be undone by executing the method in reverse, but if the band wraps around some object, say a cylinder, then the cylinder could prevent them from disentangling. Fig. 1 shows such a configuration involving two rubber bands. It's impossible for them to come apart with the cylinder in place. However, in this example, they don't form a band longer than the original.
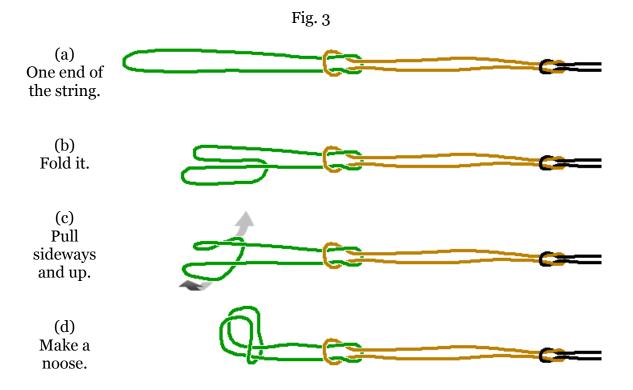

Fig. 1. A loop formed
by two rubber bands.

Probably everyone knows how to join rubber bands to make an arbitrarily long 'string' as shown in Fig. 2. But, to make a band, the two ends need to be joined. Unfortunately, there doesn't seem to be any way of joining them that's truly secure. There are ways to connect the ends together using a stick or semi-rigid object, but they all depend on friction between the rubber and the stick and on keeping the band under tension. These may be *practical* solutions, but they violate the requirements. First, the connection can be undone, if not spontaneously, then by a malicious agent. Second, while not explicitly stated,

Fig. 2. A string of rubber bands.

using anything but the rubber bands is not allowed. However, the cylinder (or object) passing through the middle of the constructed band is not considered part of the band even though it's an essential part of the problem.

In fact, it *is* possible to create a longer band that is absolutely secure as long as it remains wrapped around something. Each end of the string shown in Fig. 2 can be configured into a noose as shown in Fig. 3.

Fig. 3

(a)
One end of
the string.

(b)
Fold it.

(c)
Pull
sideways
and up.

(d)
Make a
noose.

Once a noose is made, it can be enlarged to any size, assuming that the string of rubber bands is long enough. Placing the noose at one end of the string around the cylinder is not enough. The other end of the string needs to be anchored on something. However, the only thing available is the cylinder. That's why a noose is needed at both ends of the string. The noose at the other end can be inserted over the cylinder in either one of two ways. After doing it one way, you can remove the noose that was inserted last, rotate it 180 degrees, then put it back. The one shown in Fig. 4 looks a little simpler and more symmetrical than the other way (not shown). The diameter of the cylinder shown in Fig. 4 is much greater than the one in Fig. 1 to better illustrate that the constructed band can be wrapped around objects that are too big for a single rubber band.
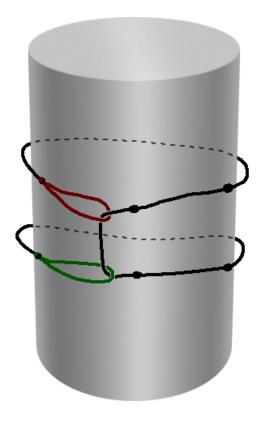
2

Fig. 4. Solution involving a noose at both ends of a string of rubber bands.

This solution involves four strands of rubber wrapping around the cylinder. A single rubber band has only one strand wrapping around. Hence, if you want to make a band that is x times longer than a single rubber band, you will need $4x$ rubber bands.

Acknowledgments

References

[1] https://franklin.dyer.me/post/144