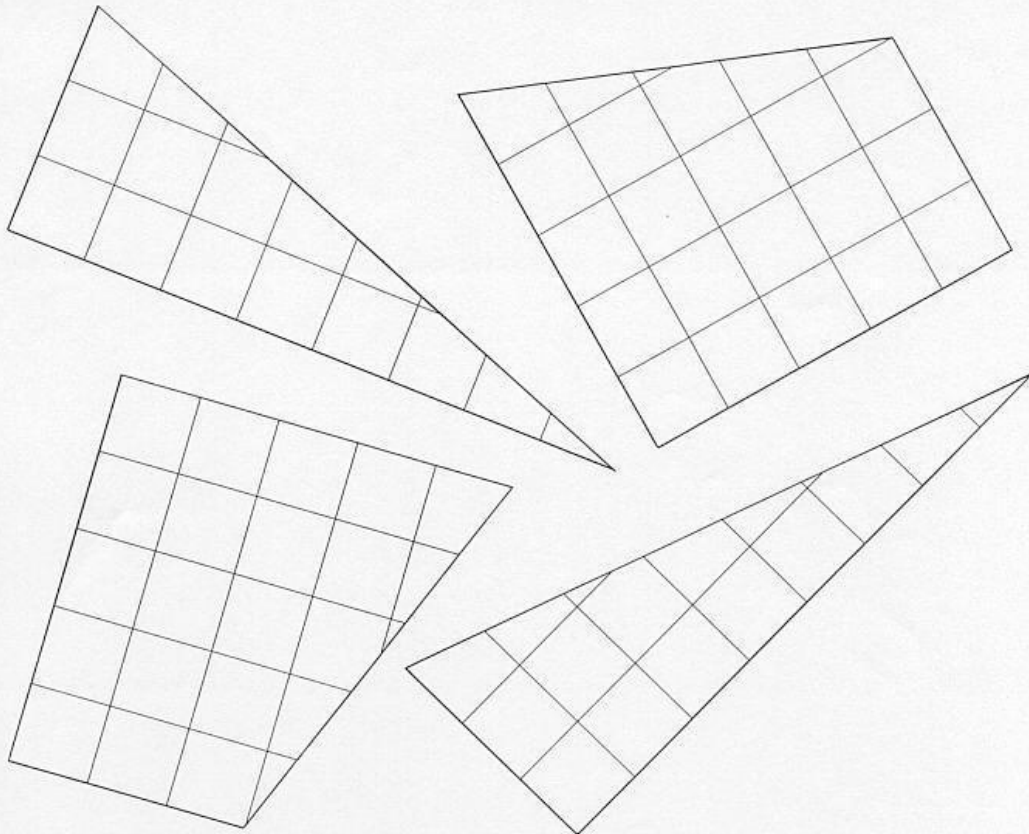


Carefully cut out the four pieces below. Put them together to make as many different shapes as you can.

Wrecked Angle

A Preview of a Paradox

Make a record of your shapes.



(1) Determine the area of each of the four original shapes.

(A) Right Triangle: _____

(B) Right Triangle: _____

(C) Trapezoid: _____

(D) Trapezoid: _____

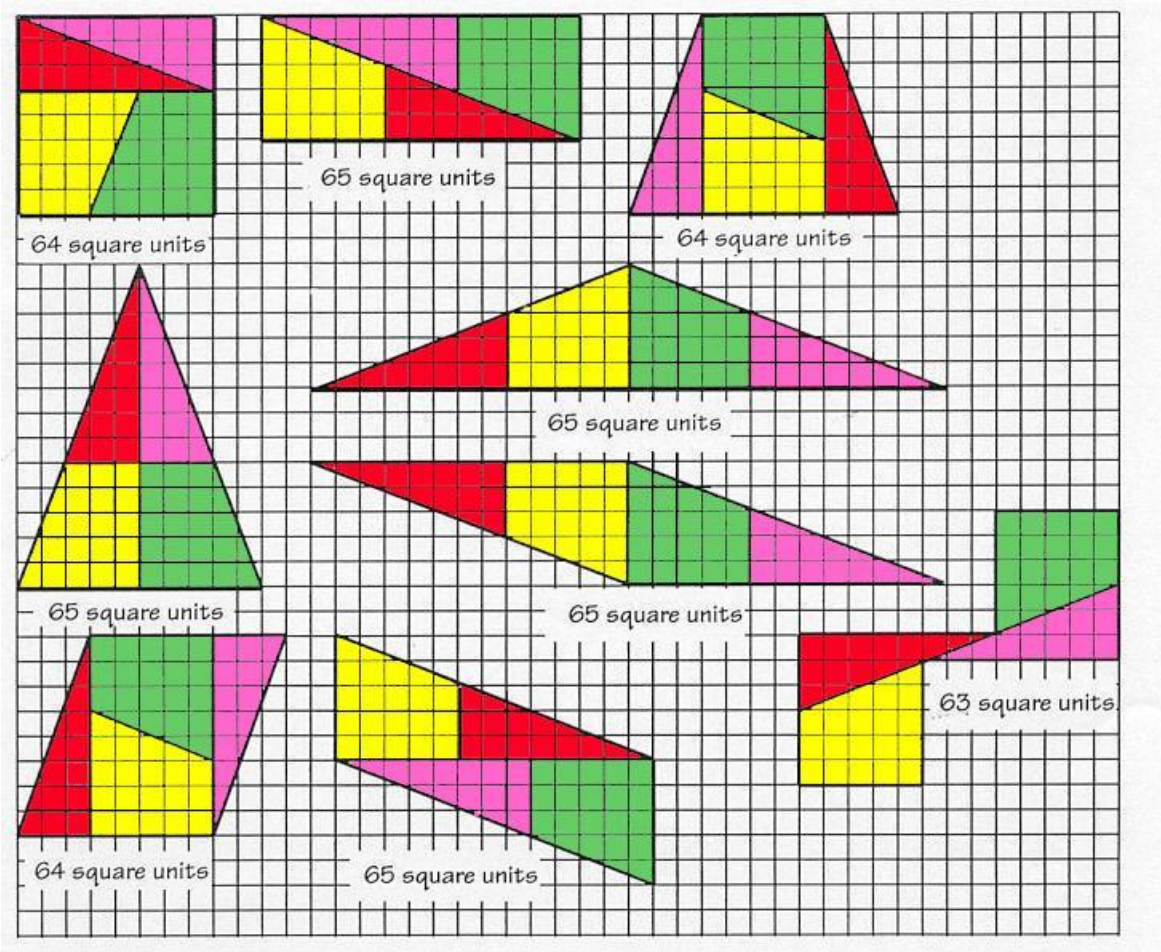
(2) Determine the area of each of the figures that you formed with the four shapes:

- (A) Square: _____
- (B) Rectangle: _____
- (C) Triangle: _____
- (D) Trapezoid: _____
- (E) Octagon: _____
- (F) Parallelogram: _____
- (G) Another Triangle: _____

There appears to be a paradox! Some of the figures have 63 square units, some have 64 square units, and some have 65 square units.

This seems paradoxical since all the figures were formed with the same four pieces.

Here are some solutions for the figures above:



How do you explain this impossible situation?

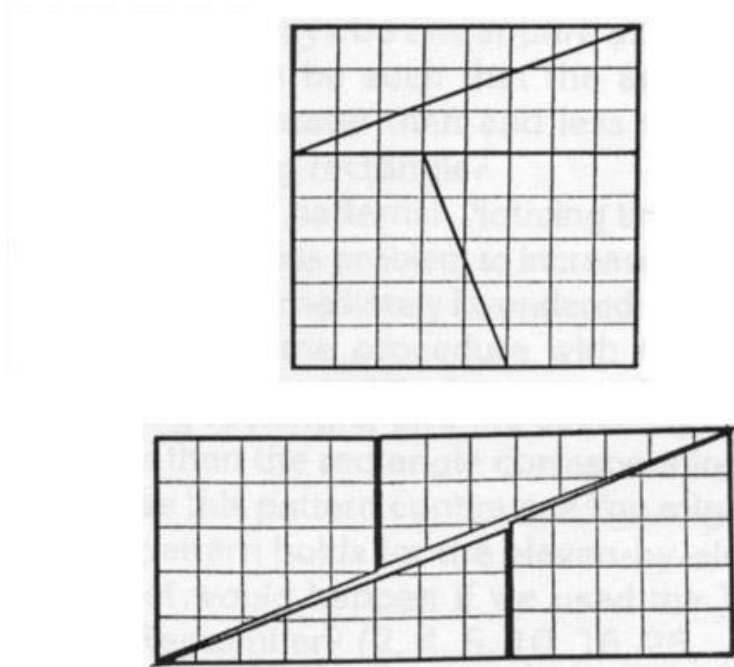
Take a closer look at the first two figures -- the square and the rectangle.

The four pieces fit snugly together for the square.

The area of the square is 64 square units, which makes sense since the areas of the four smaller pieces are 12 square units, 12 square units, 20 square units, and 20 square units.

The area of the rectangle is 65 square units, but where does that extra square unit come from?

Look at the diagram below and you will notice that the pieces do not fit snugly together in the rectangle (the extra square inch can be seen in the middle).



How can you prove this?

If the pieces indeed did fit together to form a rectangle, then the diagonal would be one line segment. If it is just one segment, then it would have just one slope, since the slope of a line is always the same. Determine the slopes of the four pieces that form "the" diagonal (the slopes should be $2/5$, $2/5$, $3/8$, and $3/8$). Notice that they are not the same. Therefore, it is not one segment (which can be seen in the drawing above).